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“Topological Algebra and Set-Theoretic Topology”

dedicated to Professor A. V. Arhangel’skii’s
80th birthday

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Alexander Vladimirovich Arhangel'skii

by Olga Sipacheva

> A good mathematician ... never grows old. He remains a child. He remains a dreamer: curious, imaginative, free of concrete purpose.
>
> —*Alexander Arhangel'skii*, interview to *The Idler* (1993)

"Arhangel'skii has a stratospheric reputation and has been regarded over the last thirty [written in 2010] years as one of the most important general and set theoretic topologists,"—said Peter Collins of Oxford University, United Kingdom, in a letter nominating Arhangel'skii for the Ohio University Distinguished Professor Award. According to other colleagues from various countries, Arhangel'skii is "one of the foremost general topologists in the world today"; his "prodigious research output is exceptional and proves him to be an original thinker and tireless author of top-quality mathematics"; he is a "great man, creator of C_p-theory"; he possesses "almost mystical intuition."

Alexander Vladimirovich Arhangel'skii was born on March 13, 1938, into a family of artists. His father, Vladimir Aleksandrovich Arhangel'skii, was a concert pianist, a student of Rachmaninoff and Igumnov and close friend of Sofronitsky, an associate professor at the Moscow Conservatory. To be more precise, Arhangel'skii Senior had begun as an aeronautical engineer and even become the first elected director of Central AeroHydrodynamic Institute, being one of the most promising students of Zhukovsky, but his great love for music had made him to change the course of his life. Arhangel'skii's mother, Mariya Pavlovna Radimova, was an acknowledged painter, a daughter of the celebrated painter and poet Pavel Aleksandrovich Radimov, the last chairman of the *Peredvizhniki* (Wanderers) society. Strange as it seems at first glance, it may be a trait inherited from parents—the ability to tangibly feel harmony and beauty—which is the mainstay of his mathematical talent and his incredible intuition. Arhangel'skii once said, "Beauty is, for me, a sign of the truth. ... When I think about a mathematical problem or theorem, my intuition suggests to me what should be true... And when I try to prove it, I will sometimes feel things fitting together harmoniously. It is from this feeling of harmony that I know my way will work." [1] In the popular science article [2] on teaching mathematics Arhangel'skii concluded several paragraphs of speculation on the harmony and beauty of mathematics with the following remarkable words: "The reader may suspect the author of likening mathematics to an art. No, mathematics *is* an art! Imagination, inspiration, and illumination are as important for mathematics as for poetry, music, and painting."

At secondary school Arhangel'skii's favorite subject was literature; he also liked biology and physics. Fortunately, in the end, he had decided to study mathematics (although he has always been and still remains a bookworm: he reads a lot in Russian, English, French, Italian, Spanish, and, very likely, some other languages). So, in 1954, he passed highly competitive entrance examinations and entered Moscow State University. During his first year, he took a (required) course in analytical geometry taught by Pavel Sergeevich Alexandroff, whose magnetic personality could not but attract him, and attended Vitushkin's introductory seminar on function theory, which provoked his interest in set theory and functions of a real variables. These two circumstances have decided his fate: when Alexandroff started an introductory seminar in set-theoretic topology, Arhangel'skii attended the very first session and decided, right during the session, to specialize in topology. Thus, Arhangel'skii the topologist was born in 1955, and Pavel Sergeevich Alexandroff, one of the most outstanding Russian mathematicians and one of the foremost pioneers of topology, had become his mentor.

In 1959 Arhangel'skii, still an undergraduate student, obtained his first important result and published his first paper, which has become the first of more than five hundred Arhangel'skii's papers. That was a metrization theorem for compact spaces based on the new notion of a network in a topological space. The result was good, but of most consequence was the introduction of the concept of a network, which is now a classical cornerstone concept in general topology. This is very typical of Arhangel'skii's work. Although he has proved numerous beautiful theorems, he has always been striving to gain insight into the essence of things rather than simply solve existing problems. It is hard to find better words than those used by Karen Shenfeld in her 1993 interview with Arhangel'skii [1]: "He has won for himself an eternal place in the history of topology, not so much through the resolution of difficult problems (though he has solved his fair share) but rather through the creation of concepts. These concepts ... have become fundamental to the ways in which topologists think about space." Some of the fundamental notions introduced by Arhangel'skii (in addition to the notion of a network) are those of tightness, free sequence, strong development, regular base, τ-monolithic space, τ-balanced, τ-bounded (τ-narrow), and τ-representable topological group, σ-paracompactness, p-space, α_i-space, cleavability, Moscow space, and so on; the list might be made very long. Arhangel'skii is also very resourceful in inventing amazingly impressive and associative names for new objects and properties: feathered, lacy, favorable, Moscow, monolithic, radial, cleavable, and so on.

The same year Arhangel'skii graduated, married, and was admitted as a postgraduate student at Moscow State University. Two years later, in 1961, Arhangel'skii participated in his first international conference, the First Symposium on General Topology in Prague; since then, he missed only two TOPOSYMs (because of health problems) and attended many dozens of other conferences, mostly as an invited speaker.

In 1962 Arhangel'skii was awarded his candidate of sciences degree (the Russian equivalent of PhD) and had to make a hard choice. At that time, his teacher, Alexandroff, headed the Department of General Topology at Steklov Mathematical Institute of the Academy of Sciences of the USSR and was the chairman of the Department of Higher Geometry and Topology at Moscow State University. Arhangel'skii had to decide where to work. He had preferred the University without any hesitation, although the Academy was more prestigious and working there did not require teaching. He has always thought that teaching students and exchanging ideas with them are of great help for research; besides, he wanted to be useful. Time has shown that, indeed, Arhangel'skii second vocation is teaching. His first PhD student was awarded degree in 1965, and his second and favorite one, Mitrofan Choban, in 1969. In all, 37 Arhangel'skii's students have been awarded PhD degree (see [3]); many of them have become doctors of science (a postdoctoral degree, Russian analogue of German habilitation) or full professors in different countries. Mitrofan Choban was his first student awarded the doctor of science degree. Even now, after more than fifty years, when something goes wrong with his *Skype*, Arhangel'skii's primary concern is communicating with Mitrofan.

Arhangel'skii's outstanding teaching ability is closely related to his unique talent for posing problems. Virtually all his talks end with dozens of problems, most of which are challenging and all interesting. Solving a problem of Arhangel'skii is a good reason for being proud. Each term several sessions of his seminar (he calls them "divertissements") are devoted entirely to problems. During these sessions, all participants are invited to pose and discuss problems, but nobody can match Arhangel'skii in this respect. He is very generous in offering his problems to students; moreover, he often abstains from thinking on a problem if he believes that it may have interesting consequences and can possibly be solved by students from the younger generation. He never assigns (but may offer when needed) particular problems to his students; instead, he encourages the students to choose the problems most interesting to them by themselves.

Arhangel'skii's problems, which are concerned with very diverse areas, along with his new concepts, have greatly influenced the whole development of topology.

Apparently, this talent for posing problems originates from Arhangel'skii's breadth of interest, which has given him a comprehensive view of topology and enabled him to see connections hidden from people investigating (maybe, very deeply) only certain special things. In his 1991 interview to *Friends of Mathematics Newsletter* of Kansas State University [4], Arhangel'skii substantiated his choice of topology as follows: "I consider topology as a fundamental subject. My ideal is to study one of the basic concepts, maybe not only of mathematics, but one could also think of it as one of the main concepts of a philosophical nature, of life, of common sense,—the concept of continuity. So if someone asks me what topology is about, I would say the main idea is to formalize, mathematically, the idea of continuity. ... Principles developed in a basic subject like algebra and topology help to view mathematics as a whole. They help to unify it. ... With those basic principles, you can try to unify mathematics to have a common language. Also, I think, general topology is very good for teaching, ... because, being very basic, it does not need other subjects for beginners. ... Another thing that is probably related is that there are many, many problems. ... The problems are at any level: simple, a little bit more difficult, more difficult, and so on. And they appear, some new ones. My teacher, Alexandroff, was saying that the most important thing in mathematics for young people is to enter the door to creating mathematics; then from that door you can move."

In 1966 Arhangel'skii became Doctor of Science. Awarding this highest academic degree in Russia at such a young age was (and still remains) quite uncommon, but Arhangel'skii had already been internationally recognized for his concept of a network and published a couple dozen papers. One of those papers, entitled *Mappings and Spaces* [5], was a long (about 50 pages) half-survey, half-research paper, which contained the first systematic reciprocal classification of spaces and maps. This paper is also remarkable in that Arhangel'skii showcased himself not only as an accomplished mathematician but also as a poet (look at the epigraph: "On the Edges of Darkness / I sing of Your Galaxies"; this is a loose translation, the Russian original reads as something like "On the branches of Darkness / Blooms the Lilac of Galaxies.")

The next year Arhangel'skii applied for membership in the Communist Party of the Soviet Union. Joining the Party disagreed with his view that the university should be free from all politics; however, in his own words, he simply recognized that, to play a public role in the life of the university at that time, he had to become a member. Indeed, many of his students, including the author of this note, would never be accepted as post-graduate students without his strong "communist" protection (according to formal rules, only the most active members of Komsomol could be accepted, while young mathematicians often considered themselves to be too decent or too busy with science for that, or simply were too careless). However, too years had passed before Arhangel'skii was admitted to the Party, because he had signed a letter of protest against the psychiatric confinement of his elder colleague Esenin-Volpin, who was a notable dissident (that was his third political imprisonment). The letter was broadcasted by *Voice of America*, and Esenin-Volpin was released almost immediately thereafter. The punishment might have been much more severe than it was (Arhangel'skii was reprimanded in oral and written form), but Arhangel'skii did not hesitate to put his signature—he "wanted to state that there were some things that [he] could not accept" [1].

In 1969 Arhangel'skii proved that the cardinality of any first-countable compact Hausdorff space does not exceed that of the continuum; thereby, he solved an almost fifty-year-old problem of Alexandroff and Urysohn. In fact, Arhangel'skii proved a much more general theorem, namely, that $|X| \leq 2^{\chi(X)L(X)}$ for any T_1-space X. His proof essentially used the new keystone notion of a free sequence introduced by him in a previous paper; as Richard Hodel mentioned in [7], an important legacy of the theorem was "the emergence of the closure method as a fundamental unifying device in cardinal functions." In [7] Hodel formulated two criteria for a theorem to be great (the theorem should solve a long-standing problem, and it must introduce new techniques and generate new results and open problems) and explains in detail why Arhangel'skii's theorem satisfies both requirements.

The first decade of Arhangel'skii's career as a university faculty member was crowned with winning the Lenin Komsomol Prize, a very prestigious state award for young scientists, engineers, and artists. It should also not be forgotten that his son was born during the same decade (he had already had a baby daughter at the beginning of his career).

The next two decades were as productive as the first. Arhangel'skii spent 1972–75 in Pakistan, at the University of Islamabad, as the "official UNESCO expert on topology" (still retaining the position at the Moscow State University), wrote hundreds of seminal papers and participated in tens of conferences (in fact, his frequent travels abroad began only with *perestroika*). In the early 1990s, the situation in Russia was quite uncertain, and in 1993 Arhangel'skii accepted professorship at Ohio University. Since then, he spent half time in Moscow and half time in Athens, Ohio, every year. In 2003 he was awarded the title of Distinguished Professor of Ohio University. However, this did not protect him from being fired, among other prominent mathematicians, from the Moscow State University for spending too much time abroad. Soon after that yet another, even worse, misfortune befell Arhangel'skii: he had lost his vision almost completely.

That was a real calamity for a man who could not live without reading. However, Arhangel'skii has never given way to despair. First, he learned to listen to audio books; then, little by little, he began to read electronic books and journal articles on a computer (huge white letters on black background). Now he again teaches at the Moscow State University (and again combines this job with working at another place, this time the Moscow Pedagogical State University), again leads a seminar, obtains beautiful results, writes papers (during the past year he published six papers), and delivers keynotes at conferences.

The contribution of Arhangel'skii to topology cannot be overestimated. Not only has he introduced fundamental concepts and posed seminal problems; he has also performed a systematic study of classes of maps in relation to topological properties of spaces; created the theory of p-spaces; investigated the class of symmetrizable spaces; proved metrization theorems; made a dominant contribution to the foundation of C_p-theory, the theory of free topological groups, and the theory of generalized topological groups (such as para-, semi-, and quasi-topological groups); developed the theories of relative topological properties, cleavable spaces, and weakly normal spaces. At present, he works extensively on topological homogeneity and remainders of compactifications.

Arhangel'skii has a highly charismatic personality. He is one of those, very few nowadays, old-school professors who has made Russian science. He is a very interesting conversationalist and can converse eloquently and competently on any subject—literature, music, politics, poetry, history, philosophy...

It seems that the most appropriate concluding words are those written by Arhangel'skii himself [2]:

"Mathematical activity is not for everybody.

"Working with abstract concepts, dealing with them as the most perfect object of real world (and this attitude is characteristic of any mathematician) are possible only for those who loves the subject. ...

Doing mathematics requires loving it."

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Abstracts of Talks

Compact complement topologies and a characterization theorem for k-spaces

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The compact complement topology of the real line was considered in the book "Counterexamples in Topology" by Steen and Seebach. In this talk, we will consider the compact complement topology of Hausdorff spaces and we will present some of its elementary properties. Moreover, we will present a characterization theorem for k-spaces.

On σ-compact groups and their mappings

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It is well known which compacta can be represented as continuous images of compact topological groups: they constitute the class of dyadic compacta. Much less is known about the compacta which are continuous images of σ-compact topological groups. In connection with the concept of a dyadic compactum, we naturally come to the next general question:

Question 1. Given a continuous mapping f of a topological group G onto a compact Hausdorff space Y, when does there exist a compact (a σ-compact) subspace X of G such that $f(X) = Y$?

If such X exists (which is, clearly, not always the case), then the compactum Y is a continuous image of a σ-compact subgroup of G, and we come to the problem mentioned above.

A result in this direction, which is easy to establish with the help of some deep facts of topological algebra and of the theory of function spaces, is this statement:

every Eberlein compactum, which is a continuous image of a σ-compact topological group, is metrizable. In particular, the free topological group of an arbitrary non-metrizable Eberlein compactum cannot be continuously mapped onto this compactum.

In this talk, we present some new results on compacta which are continuous images of σ-compact topological groups and some closely related results.

All spaces considered are assumed to be Tychonoff. A *condensation* is a one-to-one continuous mapping of one space onto some other space. Below is a typical result from the talk.

Theorem 1. *Suppose that a σ-countably compact semitopological group G condenses onto a sequential Dieudonné complete space Y with the Baire property. Then G is a separable metrizable locally compact σ-compact topological group (and Y has a countable network).*

A new concept of homogeneity type is introduced and applied to the study of free topological groups. A space X is said to be *k-homogeneous* if every compact subspace of X is contained in a homogeneous compact subspace. It is shown that not every σ-compact topological group is k-homogeneous.

On the Mackey topology on abelian topological groups

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For a locally convex vector space (V, τ) there exists a finest locally convex vector space topology μ such that the topological dual spaces $(V, \tau)'$ and $(V, \mu)'$ coincide algebraically. This topology is called *Mackey topology.* If (V, τ) is a metrizable locally convex vector space, then τ is the Mackey topology.

In 1995 Chasco, Martín Peinador and Tarieladze asked the following question: Given a locally quasi-convex group (G, τ), does there exist a finest locally quasi-convex group topology μ on G such that the character groups $(G, \tau)^\wedge$ and $(G, \mu)^\wedge$ coincide?

In this talk we give examples of topological groups which

1) have a Mackey topology,

2) do not have a Mackey topology,

and we characterize those abelian groups which have the property that every metrizable locally quasi-convex group topology is Mackey (i.e. the finest compatible locally quasi-convex group topology).

On dense subspaces of iterated function spaces

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In the paper [Baturov, *Moscow Univ. Math. Bull.*, 1988] it is proved that if a Tychonoff space X has a point-countable network of cardinality $\aleph_1$ and Y is a normal dense subspace of $C_p(X)$, then Y is collectionwise normal. We will discuss some analogous results for dense subspaces of $C_{p,n}(X)$.

On a game-theoretic version of Arhangel'skiĭ's inequality

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We present a cardinal inequality for a space with points G_δ, obtained with the help of a long version of the Menger game. This result improves a similar one established by Scheepers and Tall.

⋆ This is a joint work with Leandro F. Aurichi (University of Sao Paolo, Brazil).

Some constructions involving inverse limits

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We will describe some inverse limit constructions of irreducible preimages of zero-dimensional compact Hausdorff spaces with some additional properties.

On g-barrelled groups

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Barrelled spaces constitute a well behaved class of locally convex spaces. They were introduced by Bourbaki in 1950 and its main feature is that they are exactly the class of spaces which satisfy the Closed Graph Theorem. If X denotes a locally convex space, X is barrelled if and only if every $w(X^*, X)$-bounded subset $M \subset X^*$ is equicontinuous.

If (G, τ) denotes an abelian topological group, call $G^\wedge := \mathrm{CHom}(G, \mathbb{T})$ the group of all the continuous characters of G, and let τ_p denote the topology of pointwise convergence on $G^\wedge$.

An abelian topological group (G, τ) is *g-barrelled* if any τ_p-compact subset $M \subset G^\wedge$ is equicontinuous with respect to τ. The g-barrelled locally quasi-convex groups constitute a subclass of groups for which there is valid an analogue of the Mackey–Arens theorem stated in terms of groups. Every barrelled vector space is a g-barrelled group.

An abelian topological group (G, τ) has the EAP_w if every τ_p-continuous arc of $(G^\wedge, \tau_p)$ is equicontinuous. Clearly, every g-barrelled group has the EAP_w.

We will present some results about these two important properties g-barrelledness and EAP_w including its preservation through products, direct sums and subgroups.

$\star$ This is a joint work with T. Borsich, X. Domínguez, and E. Martín-Peinador.

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Complete normal bases

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A compactum cX is a *compactification* of Tychonoff space X, if $c\colon X \to cX$ is a homeomorphic embedding of X into cX and $\overline{c(X)} = cX$. For compactifications c_1X and c_2X assume $c_2X \geq c_1X$, if there exists a continuous mapping $f\colon c_2X \to c_1X$ such that $f \circ c_2 = c_1$. For a compactification cX, let $\mathcal{A}$ be a set of all continuous functions on X continuously extended over cX. Then a family $\mathcal{F} = \{f^{-1}(0) : f \in \mathcal{A}\}$ forms a normal base, and for the Wallman compactification $\omega(X,\mathcal{F})$ it takes place $\omega(X,\mathcal{F}) \geq cX$ [3]. If $\omega(X,\mathcal{F}) = cX$ then the compactification cX is called a *β-like compactification* [6]. For a β-like compactification $cX = \omega(X,\mathcal{F})$, a family $\mathcal{F}$ is a separating nest-generated intersection ring (s.n.-g.i.r.) [5] and $\mathcal{A}$ is an inversion-closed algebra [2, 4, 7]. Points of β-like compactification cX are all ultrafilters of $\mathcal{F}$. Part $\upsilon(X,\mathcal{F})$ of β-like compactification cX is the set of all CIP (countable intersection property) ultrafilters of $\mathcal{F}$ in induced topology from $\omega(X,\mathcal{F})$ is called a *Wallman realcompactification* of X [5].

Let $\tau \geq \aleph_0$ be an arbitrary cardinal. An ultrafilter p of $\mathcal{F}$ is called *τ-co-locally finite* (τ-co-LF), if $p' \subset p$ whenever $|p'| \leq \tau$ and $Cp' = \{X \setminus Z : Z \in p'\}$ is locally finite, then $\cap p' \neq \emptyset$. Denote as $\mu_\tau(X,\mathcal{F})$ the set of all τ-co-LF ultrafilters of $\mathcal{F}$ on X. Note that every τ-co-LF ultrafilter is CIP. Hence, $\mu_\tau(X,\mathcal{F}) \subseteq \upsilon(X,\mathcal{F}) \subseteq cX = \omega(X,\mathcal{F})$ for any $\tau \geq \aleph_0$. Note that $\mu_\tau(X,\mathcal{F}) = \upsilon(X,\mathcal{F})$ whenever $\tau = \aleph_0$.

A Tychonoff space X is called *τ-topologically complete* with respect to the s.n.-g.i.r.-base $\mathcal{F}$, if every τ-co-LF ultrafilter of $\mathcal{F}$ converges. A space X is dense in $\mu_\tau(X,\mathcal{F})$, and $\mu_\tau(X,\mathcal{F})$ in induced topology from $\omega(X,\mathcal{F})$ is a τ-topological completion of X with respect to the s.n.-g.i.r.-base $\mathcal{F}$ and $\mu_\tau(X,\mathcal{F})$ is topologically complete or Dieudonné complete for any $\tau \geq \aleph_0$. For uniform spaces the same tasks were solved in [7, 8].

A s.n.-g.i.r.-base $\mathcal{F}$ on a space X is said to be τ*-complete* if it coincides with the family $\hat{\mathcal{F}} = \{Z \cap X : Z \in \mathcal{Z}(\mu_\tau(X, \mathcal{F}))\}$ and whenever $\tau = \aleph_0$, τ-complete s.n.-g.i.r.-base coincides with complete base in sense of J. L. Blasco [1]. Since $\mathcal{F}$ is the trace on X of all zero-sets in the Wallman compactification $\omega(X, \mathcal{F})$, we have $\mathcal{F} \subset \hat{\mathcal{F}}$. An example of a complete base is $\mathcal{Z}(X)$.

In this talk, some properties of τ-complete s.n.-g.i.r.-bases are investigated. In particular,

- If $\mathcal{F}$ is a s.n.-g.i.r.-base on a space X, then $\mu_\tau(X, \hat{\mathcal{F}}) = \mu_\tau(X, \mathcal{F})$.
- A s.n.-g.i.r.-base $\mathcal{F}$ on a space X is τ-complete if and only if the Čech–Stone compactification $\beta(\mu_\tau(X, \mathcal{F}))$ coincides with the Wallman compactification $\omega(X, \mathcal{F})$.

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Conditions of finiteness, open mappings and classes of spaces

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Any space is considered to be a T_0-space. We use some notions from [1, 2, 3, 4]. We study the following general question: *Under which conditions the bounded subsets of the given space are finite?*

A subset A of X is *bounded* if for every locally finite family γ of open non-empty subsets in X the set $\{U \in \gamma : U \cap A \neq \emptyset\}$ is finite.

A space X is called a *feebly compact* space if every locally finite family γ of open non-empty subsets in X is finite.

A space X is called an *F-space* if X is completely regular and if disjoint cozero-sets of X are contained in disjoint zero-sets.

A space X is called an *F'-space* if X is completely regular and $\mathrm{cl}_X U \cap \mathrm{cl}_X V = \emptyset$ for every two disjoint functionally open sets U and V of X.

A *Maltsev polyalgebra* is a space G with one ternary compact-valued upper semicontinuous set-valued operation $m_G \colon G^3 \to G$ such that $m_G(y,x,x) = m_G(x,x,y) = \{y\}$ for all $x, y \in G$. The mapping m_G is called a *Maltsev polyoperation.* If the mapping m_G is single-valued, then G is a *Maltsev algebra.* A mapping $\varphi \colon A \to B$ of a Maltsev polyalgebra A into a Maltsev polyalgebra B is a homomorphism if $\varphi(m_A(x,y,z)) \subset m_B(\varphi(x), \varphi(y), \varphi(z))$ for all $x, y, z \in A$.

Our aim is to present some results of the following kind.

Theorem 1. *Let G be a topological Maltsev polyalgebra. Then G is a T_2-space.*

Theorem 2. *Let $\varphi \colon A \to B$ be a quotient homomorphism of a Maltsev polyalgebra A onto a Maltsev polyalgebra B, and (G, m) be a topological Maltsev polyalgebra. Then φ is an open homomorphism.*

Theorem 3. *Let G be a compact Maltsev polyalgebra. Then for any continuous mapping $f \colon G \to Y$ onto an infinite Hausdorff space Y there exist a compact Maltsev polyalgebra B, a continuous open homomorphism $\varphi \colon G \to B$ of G onto B and a continuous mapping $g \colon B \to Y$ such that $w(B) \leq w(Y)$ and $f = g \circ \varphi$.*

Corollary 4. *Let G be a compact Maltsev polyalgebra. If G is an F'-space of poitwise countable type, then G is a discrete space.*

Corollary 5 (see [4]). *Let G be a compact space. A space G admits a structure of a compacr polyalgebra if and only if G is a Dugundji space.*

Theorem 6. *For any non-empty Tychonoff space X there exists a compact Maltsev polyalgebra $G(X)$ such that:*

- *X is a subspace of $G(X)$ and $G(X) = H$ provided that $X \subset H \subset G(X)$ and H is a closed Maltsev subpolyalgebra of $G(X)$;*
- *for any continuous mapping $f \colon X \to A$ into a compact Maltsev polyalgebra A there exists a continuous homomorphism $\varphi \colon G(X) \to A$ such that $f = \varphi|X$.*

Theorem 7. *If G is a Maltsev algebra and an F'-space, then any compact subset of G is finite.*

Theorem 8. *Let H be a closed subgroup of a topological group G and $X = G/H$ be the quotient space. If a topological space X is an F'-space, then any bounded subset of X is finite.*

Theorem 9. *Let G be a pseudocompact Maltsev algebra. If G is an F-space, then the space G is finite.*

Some open questions will be formulated.

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P-points, Silver reals, and Isbell's problem

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P-ultrafilters are topological P-points in the space of non-principal ultrafilters on natural numbers. S. Shelah proved that the existence of P-ultrafilters in not provable in ZFC. I will present an overview of a recent alternative method of demonstrating that fact. Applications of this method yield interesting results, it is e.g. possible to get a model of with no P-points and the continuum arbitrarily large, and to prove that certain inequality between cardinality between cardinal invariants implies non-existence of P-points. I will also mention a possible connection with Isbell's problem, which concerns the existence of Tukey non-equivalent ultrafilters.

⋆ These results are joint work with Osvaldo Guzmán and Jonathan Verner.

Extensions of dualities and a new approach to Fedorchuk's duality

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We prove a general categorical theorem about extensions of dualities. Using it, we obtain a new proof of Fedorchuk's Duality Theorem [V. V. Fedorchuk, *Sibirsk. Mat. Ž.*, 1973; English translation: *Siberian Math. J.*, 1974].

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Metrization of the space of weakly additive functionals

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Let X be a metric space. Denote by $C_b(X)$ the algebra of all real-valued bounded continuous functions on X. A functional $\nu\colon C_b(X) \to R$ is said to be

- *weakly additive* if $\nu(\varphi + c_X) = \nu(\varphi) + c\nu(1_X)$ for all $\varphi \in C_b(X)$ and $c \in \mathbb{R}$;
- *order-preserving* if for any $\varphi, \psi \in C_b(X)$ with $\varphi \le \psi$ we have $\nu(\varphi) \le \nu(\psi)$;
- *normalized* if $\nu(1_X) = 1$;
- *positively homogeneous* if $\nu(t\varphi) = t\nu(\varphi)$ for all $\varphi \in C_b(X)$, $t \in \mathbb{R}$, $t \ge 0$.

For a metric space X we denote by $OH(X)$ the set of all weakly-additive, order-preserving, normalized and positively homogeneous functionals on $C_b(X)$. Set

$$\mathrm{Lip}_1(X) = \{\varphi\colon X \to R, |\varphi(x) - \varphi(y)| \le \rho(x,y),\ \forall\, x,y \in X\}.$$

Let us define a Kantorovich–Rubinshtein metric on $OH(X)$ by

$$\rho_{OH}(\mu,\nu) = \sup\{|(\mu-\nu)(\varphi)| : \varphi \in \mathrm{Lip}_1(X)\},\ \ \mu,\nu \in OH(X). \tag{1}$$

We prove the following

Theorem. *The function ρ_{OH} defined by* (1) *is a metric on $OH(X)$.*

Convergence methods in topology: Recent examples

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A convergence τ is a *topology* whenever $\mathrm{T}\tau \geq \tau$, where T stands for the topologizer, a concrete reflector in the category of convergences with continuous maps as morphisms.

Several other (concrete) reflective subcategories are characterized analogously, for example, pretopologies (S_0), paratopologies (S_1), pseudotopologies (S), while (concrete) coreflective subcategories are characterized by inequalities of another type, for example, a convergence ξ is of *countable character* if $\xi \geq \mathrm{I}_1\xi$, where I_1 is a certain concrete coreflector. Moreover, many fundamental subclasses of topologies admit characterizations in terms of *functorial inequalities* of the type

$$\tau \geq \mathrm{JE}\tau, \tag{1}$$

where J is a reflector and E is a coreflector. For instance, *sequential* topologies τ ($\tau \geq \mathrm{TI}_1\tau$), *Fréchet* topologies τ ($\tau \geq \mathrm{S}_0\mathrm{I}_1\tau$), *strongly Fréchet* topologies τ ($\tau \geq \mathrm{S}_1\mathrm{I}_1\tau$), *bisequential* topologies τ ($\tau \geq \mathrm{SI}_1\tau$).

Continuity can be characterized in terms of final and initial convergences (a map $f\colon |\xi| \to |\tau|$ is continuous if and only if $f\xi \geq \tau$, equivalently, $\xi \geq f^-\tau$). Classical variants of *quotient maps* are characterized by *functorial inequalities* of the type

$$\tau \geq \mathrm{J}\left(f\xi\right), \tag{2}$$

where J is a reflector, for instance, T for *(topological) quotient maps,* S_0 for *hereditarily quotient maps,* S_1 for *countably biquotient maps*, S for *biquotient maps.* These characterizations enables us to easily infer about preservation of properties of the type (1) by maps of the type (2).

Variants of *compactness* can be characterized in terms of subcategories of convergences; various types of *perfect maps* they can be described as inversely preserving of certain types of compactness Finally, functorial inequalities applied to products of spaces and of functions, enable us to study various productivity quests.

I will illustrate these techniques on an example of a recent characterization of *productively sequential* topologies [S. Dolecki, F. Mynard, *Math. Slovaca*, 2018].

Theorem 1. *A topology is productively sequential if and only if its product with each strongly sequential topology (equivalently, strongly Fréchet topology) is sequential (equivalently, strongly sequential).*

Theorem 2. *A regular topology is productively sequential if and only if it is sequential and bi-quasi-k.*

Such topologies can be characterized by a functorial inequality.

Fixed points and coincidences of mappings and mapping families in ordered sets

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The report is devoted to the fixed point and coincidence existence problems for mappings of partially ordered sets. Recently, in the papers [D. A. Podoprikhin, T. N. Fomenko, *Doklady Math.*, 2016] and [T. N. Fomenko, D. A. Podoprikhin, *Topol. Appl.*, 2017] fixed point and coincidence existence theorems were presented for mapping families of ordered sets. In the report we give some generalizations of that results.

Given a metric space (X, d), one can construct an ordered set $(X, \preceq_\varphi)$ where the order $\preceq_\varphi$ is defined as follows. For any $x, y \in X$, we say $x \preceq_\varphi y \iff d(x,y) \leq \varphi(y) - \varphi(x)$. We call this order *Brøndsted order*. It was introduced in the paper [A. Brøndsted, *Pacific J. Math.*, 1974]. Using such kind of the ordering a given metric space, we discuss connections between the obtained recent results in ordered sets and some known metrical results concerning fixed point and coincidence problems. For example, some generalizations of the well-known Caristi fixed point theorem, to the case of a set-valued mapping and also to the case of coincidences of mappings, are obtained. The results of the papers [T. N. Fomenko, *Moscow Univ. Math. Bull.*, 2017], [T. N. Fomenko, *Doklady Math.*, 2017] and some further developments of those are discussed in the talk.

In addition, we touch upon the connection between the cascade search method of finding zeros of functionals in a metric space (see, for example, the papers [T. N. Fomenko, *Topol. Appl.*, 2010], [T. N. Fomenko, *Math. Notes*, 2013]) on the one hand, and some facts of the theory of partially ordered sets, on the other hand.

An Antoine Necklace in $\mathbb{R}^3$ all whose projections are 1-dimensional

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L. Antoine constructed a Cantor set in $\mathbb{R}^2$ whose projections coincide with those of a regular hexagon [1, **9**, p. 272; and fig. 2 on p. 273].

By K. Borsuk [3], there exists a Cantor set in $\mathbb{R}^d$ such that its projection onto every hyperplane contains a $(d-1)$-dimensional ball, or equivalently, has dimension $(d-1)$.

J. Cobb [4] gives an example of a Cantor set in $\mathbb{R}^3$ such that its projection onto every 2-plane is 1-dimensional (for higher-dimensional extensions, see [5, 2]). Cobb's method is rather sophisticated; the resulting Cantor set is tame.

Interestingly, there exists an easily described (wild) Cantor set in $\mathbb{R}^3$ all whose projections are 1-dimensional. This is a well-known Antoine Necklace.

Theorem 1. *There exists an Antoine Necklace A in $\mathbb{R}^3$ such that for each 2-dimensional plane $\Pi \subset \mathbb{R}^3$, we have $\dim p_\Pi(A) = 1$.*

(Here $p_\Pi \colon \mathbb{R}^3 \to \Pi$ is the orthogonal projection onto a 2-dimensional plane $\Pi \subset \mathbb{R}^3$.)

Bing–Whitehead Cantor sets [6] are wild Cantor sets of other type: in contrast to Antoine necklaces, they have simply connected complement.

Theorem 2. *There exists a Bing-Whitehead Cantor set B in $\mathbb{R}^3$ such that for each 2-dimensional plane $\Pi \subset \mathbb{R}^3$, we have $\dim p_\Pi(B) = 1$.*

Our method gives *uncountably many* pairwise ambiently non-equivalent Antoine necklaces and Bing–Whitehead Cantor sets in $\mathbb{R}^3$ all whose projections are 1-dimensional.

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On embedding of spaces into Tychonoff products

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The A. V. Archangel'skiĭ theorem on the cardinality of a compact Hausdorff first countable space caused a great increase in the study of cardinal characteristics of spaces and methods of their proofs.

We consider problems of an embedding of a space X into Tychonoff products of spaces, which has some properties of projections.

Using this embedding, we study properties of the space X as a subspace of the product and prove the cardinal characteristics of X that follow from these properties.

On the classification of spaces $C_p(X)$, where X is a countable metric space

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Our talk will be devoted to problems of uniform and linearly topological classification of spaces $C_p(X)$, where X is a countable metric space.

When a totally bounded group topology is the Bohr Topology of a LCA group

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We look at the Bohr topology of maximally almost periodic groups (MAP, for short). Among other results, we investigate when a totally bounded abelian groups (G, w) is the Bohr reflection of a locally compact abelian group. Necessary and sufficient conditions are established in terms of the inner properties of w. As an application, an example of a MAP group (G, t) such that every closed, metrizable subgroup N of bG with $N \cap G = \{0\}$ preserves compactness but (G, t) does not strongly respects compactness is given. Thereby, we respond to two open questions by Comfort, Trigos-Arrieta, and Ta-Sun Wu.

Factorizations of maps on limits of inverse systems

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Let $\mathcal{C}$ be an epireflective subcategory of Hausdorff objects of topological or uniform spaces generated by a class $\mathcal{A}$. For a cardinal λ, we say that $\mathcal{C}$ has property F_λ if every $\mathcal{C}$-morphism into any object from $\mathcal{A}$ defined on a limit of an inverse system in $\mathcal{C}$ factorizes via a limit of a subsystem of cardinality less than λ. The problem of finding such λ comes from investigation of locally presentable categories and was considered in the paper [M. Hušek, J. Rosický, "Factorization and local presentability in topological and uniform spaces," *submitted*, 2018].

In topological spaces, the situation is easy for classes of compact spaces and not easy for realcompact spaces (taking reals for the generator). For that and similar cases, large cardinals are needed.

Theorem 1. *The class of Hausdorff realcompact spaces has F_λ for some λ if ω_1-strongly compact cardinals exist. For λ one may take the least regular ω_1-strongly compact cardinal.*

The last result generalizes to Herrlich's τ-compact spaces—one assumes the existence of τ-strongly compact cardinals. Similar results hold for some complete proximity spaces and for Dieudonné complete spaces. Other classes will be considered, too.

Existence of measurable cardinals is necessary for Theorem 1 to hold but we do not know if the existence of ω_1-strongly compact cardinals is necessary. But at least in some situations the existence of measurable cardinals is not enough.

Almost fully closed mappings and quasi-F-compacta

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We introduce the notions of an almost fully closed mapping and a quasi-F-compactum (all spaces are Hausdorff compact), which generalize the concepts of a fully closed mapping and a Fedorchuk compactum.

Definition 1. A continuous mapping $f\colon X \to Y$ is *almost fully closed* if for any two disjoint closed subsets $A, B \subset X$ $|f(A) \cap f(B)| \leq \omega_0$.

Definition 2. A space X is called a *quasi-F-compactum* if it is the limit of a continuous well-ordered inverse system $S = \{X_\alpha, \pi^\alpha_\beta : \alpha, \beta < \gamma\}$ where X_0 is a point, all neighboring projections $\pi^{\alpha+1}_\alpha$ are almost fully closed and all fibres $(\pi^{\alpha+1}_\alpha)^{-1}(x)$ are metrizable. The smallest length γ of such system S is called the *spectral height* of X.

The composition of almost fully closed mappings of first countable compacta is almost fully closed. All projections π^α_β and limiting projections π_α of the system S above are almost fully closed if $\gamma \leq \omega_1$.

We obtained the following results.

Theorem 1. *Let Y be a quasi-F-compactum of spectral height 3. Then, for any uncountable metrizable compactum K, the product $Y \times K$ is not a quasi-F-compactum of a countable spectral height.*

Theorem 2. *The product of quasi-F-compacta of spectral height 3 is never a quasi-F-compactum of a countable spectral height.*

Theorem 3. *There exists a perfectly normal compactum X for which any almost fully closed mapping to a metric compactum is constant.*

Vietoris-type topologies on hyperspaces

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We study the $(\mathcal{F}, \mathcal{G})$-*hit-and-miss topology* introduced by Clementino and Tholen [M. M. Clementino, W. Tholen, *Topology Proc.*, 1997]. We call it *Vietoris-type hypertopology* (since under this name it was introduced independently in [E. Ivanova-Dimova, *arXiv*:1701.01181, 2017]). We show that the Vietoris-type hypertopology is, in general, different from the Vietoris topology. Also, some of the results of Michael [E. Michael, *Trans. Amer. Math. Soc.*, 1951] about hyperspaces with Vietoris topology are extended to analogous results for hyperspaces with Vietoris-type topology. We obtain, as well, some results about hyperspaces with Vietoris-type topology which concern some problems analogous to those regarded by Schmidt [H.-J. Schmidt, *Math. Nachr.*, 1981].

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On the tightness of G_δ-modifications

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The G_δ-modification X_δ of X is generated by the G_δ subsets of X. Bella and Spadaro asked in [*arXiv*:1707.04871]: Is $t(X_\delta) \le 2^{t(X)}$ true for every (compact) T_2 space X? We answered both questions:

Theorem 1. *If X is a regular Lindelöf space then $t(X_\delta) \le 2^{t(X)}$.*

In [Juhász, Soukup, Szentmiklóssy, *Topol. Appl.*, 1998] we proved the consistency of "if X is a countably tight compactum then $t(X_\delta) \le \omega_1$" with the continuum being arbitrarily large.

Problem 1. Is it consistent to have a countably tight compactum X for which $t(X_\delta) > \omega_1$?

Theorem 2. *If there is a non-reflecting stationary set of ω-limits in a regular cardinal κ then there is a T_5 Fréchet–Urysohn topology τ on $\kappa + 1$ such that $t(\tau_\delta) = \kappa$.*

Problem 2. Is there in ZFC a countably tight space X with $t(X_\delta) > 2^\omega$?

Theorem 3. *If λ is a strongly compact cardinal then $t(X) < \lambda$ implies $t(X_\delta) \le \lambda$ for every topological space X.*

Note that (i) if there is a non-reflecting stationary set of ω-limits in κ then κ is less than the first strongly compact cardinal λ_0; (ii) modulo some large cardinals, in some ZFC model λ_0 exists and the cardinals κ admitting a non-reflecting stationary set of ω-limits are cofinal in λ_0. This means that Theorem 2 is, in some sense, sharp.

$\star$ This is a joint work with A. Dow, L. Soukup, Z. Szentmiklóssy and W. Weiss.

Fragmentability of spaces: a valuable alternative for metrizability

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Metrizability provides a very convenient topological framework for many studies in analysis. However, there are many non-metrizable topological spaces which are important for functional analysis. The most famous examples are the weak topology of an infinite dimensional Banach space as well as the weak* topology of its dual. Similarly, the pointwise convergence topology in the space $C(T)$ of all continuous functions in a non-countable compact space T is also not metrizable. As a way out, J. E. Jayne and C. A. Rogers introduced in [*Acta Math.*, **155** (1985), 41–79] a more general notion which has some of the properties of metrizability and is much more often to meet: the topological space X is said to be *fragmentable* if there exists some metric d in it (not necessarily generating its topology) such that for every non-empty set A of X and for every $\epsilon > 0$ there exists an open set V such that $V \cap A \neq \emptyset$ and the d-diameter of $V \cap A$ is less than or equal to ϵ.

The goal of this talk is to overview some of the applications of fragmentability and to present a generalization of this notion which is obtained when in the above definition one considers not all non-empty subsets A of X but only the open subsets of X. Such spaces X are called *spaces with fragmentable open sets* or, simply, *fos-spaces*.

Theorem 1 (M. M. Choban, P. S. Kenderov and J. P. Revalski)**.** *Let X be a regular topological space. Then the following properties are equivalent:*

(i) *X is a fos-space;*

(ii) *there is a sequence $\{\gamma_n\}_{n\geq 1}$ of families of disjoint nonempty open sets in X such that:*

 (1) *for every $V_{n+1} \in \gamma_{n+1}$ there exists some $V_n \in \gamma_n$ such that $V_{n+1} \subset V_n$ (γ_{n+1} is a refinement of γ_n);*

 (2) *for every $n \geq 1$ the open set $W_n := \bigcup\{V_n : V_n \in \gamma_n\}$ is dense in X;*

 (3) *if $(V_n)_{n\geq 1}$ is a sequence of sets such that $V_n \in \gamma_n$ for every $n \geq 1$ and $(V_n)_n$ is nested (i.e. $V_{n+1} \subset V_n$ for each $n \geq 1$) then the intersection $\bigcap_n V_n$ is either empty or a singleton.*

(iii) *X is the union of two disjoint sets X_1 and X_2 such that X_1 is of the first Baire category in X, and there exists a metrizable topology on X_2 which is coarser than the topology inherited from X (one of the two sets can be empty).*

If the metric d which fragments the open sets of X generates a topology which is finer than the original topology of X, then the space X_2 from item (iii) is metrizable.

Further, a topological game characterization of fos-spaces is given which allows us to show that

Theorem 2. *A compact topological space X is a fos-space if, and only if, it contains a dense completely metrizable subset.*

Global invariants of continuous paths for sinear similarity groups in the two-dimensional Euclidean space

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Global G-invariants of continuous paths in the two-dimensional Euclidean space E_2 for the linear similarity group $G = \mathrm{LSim}(2)$ and the orientation-preserving linear similarity group $G = \mathrm{LSim}^+(2)$ are studied. Complete systems of global G-invariants of continuous paths are obtained for groups $G = \mathrm{LSim}(2), \mathrm{LSim}^+(2)$. Conditions of the global G-equivalence of continuous paths are given in terms of complete systems of global G-invariants of continuous paths. General evident form of a planar continuous path with given complete systems of G-invariants is obtained. For given two planar continuous paths $x(t)$ and $y(t)$ with common complete system of G-invariants, evident forms of all transformations $g \in G$, carrying $x(t)$ to $y(t)$, are obtained.

Some results in selection principles theory

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We discuss some known and some new results in selection principles theory. In particular, we will point out influence of the contributions of A. V. Arhangel'skii to our research in this theory. A few open problem will be also considered.

On some topological property of a space $C_p(X, S)$, where S is the Sorgenfrey line

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We explore the question of the existence of the property of being an angelic space for space $C_p(X, S)$, where S is the Sorgenfrey line.

Theorem 1. *Let K be a compact space and S be a Sorgenfrey line, and $C(K, S)$ be endowed with the pointwise topology. Let A be a relatively countably compact in $C_p(K, S)$ and ψ be an element of the closure $\overline{A}$ of A in S^K. Then there is an x in $C_p(K, S)$ and a sequence x_n in A such that $x_n(t) \to x(t) = \psi$.*

Theorem 2. *Let K be a compact space, S be a Sorgenfrey line, and A be a relatively countably compact subset in $C_p(K, S)$. Then the properties of relative compactness and relative countable compactness for subsets of $C_p(K, S)$ coincide.*

From these two theorems it follows that the space $C_p(K, S)$ is angelic for the compactum K and the Sorgefrey line S.

On the dimension of some semi-metric spaces

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In dimension theory, the concept of a network introduced by A. V. Arkhangelskii plays a large role as the properties of paracompact σ-spaces [1].

A system $\mu = \{F_\alpha\}$ of sets in a space X is called a *network* if, given any point $x \in X$ and any neighborhood Ox of x, there exists an $F_\alpha \in \mu$ for which $x \in F_\alpha \subseteq Ox$.

Definition 1 [2, 3]**.** We say that a closed network $\gamma = \{F_\alpha\}$ in a space Y is an *S-network* if, given any closed set $F \subseteq Y$ and any its open neighborhood OF, there exists a subsystem $\gamma(F)$ of γ such that the union of all elements of $\gamma(F)$, i.e., $\bigcup \gamma(F)$ is closed in Y and $F \subseteq \bigcup \gamma(F) \subseteq OF$.

All topological spaces considered in this report are assumed, unless otherwise stated, to be normal, Hausdorff, and finite-dimensional. We also assume that all maps of spaces are continuous and all networks consist of closed sets.

Definition 2. A space X is *semimetrizable* if there exists a real-valued function d on $X \times X$ such that (a) $d(x, y) = d(y, x)$, (b) $d(x, y) = 0$ if and only if $x = y$, and (c) a point $p \in X$ is in the closure of a subset B of X if and only if $\inf\{d(p, b) : b \in B\} = 0$ (i.e., d generates the topology of X). The function d is called a *semi-metric*. A space X is 1-continuous semimetrizable if d is continuous in one variable.

Theorem 1. *Let X be a 1-continuous semimetrizable paracompact σ-space, let X have a σ-closure-preserving S-network. Then the following conditions are equivalent:*

(a) $\dim X \leq n$;

(b) $\operatorname{Ind} X \leq n$;

(c) $X = \bigcup X_i$, $i = 0, 1, 2, \ldots, n+1$, *where each set* X_i *is* G_δ *and* $\dim X_i \leq 0$, *for* $i = 1, 2, \ldots, n + 1$;

(d) *the space X is a $\leq(n+1)$-to-one image of a zero-dimensional semimetrizable paracompact σ-space under a perfect map.*

Theorem 2. *Let a space X be a Nagata space (i.e., stratifiable and semimetrizable) and let the semi-metric in X be 1-continuous. Then the following conditions are equivalent:*

(a) $\dim X \leq n$;

(b) $\operatorname{Ind} X \leq n$;

(c) $X = \bigcup X_i$, $i = 0, 1, 2, \dots, n+1$, *where each set* X_i *is* G_δ *and* $\dim X_i \leq 0$, *for* $i = 1, 2, \dots, n+1$;

(d) *the space* X *is a* $\leq(n+1)$*-to-one image of a zero-dimensional Nagata space under a perfect map.*

From Theorem 2 (d) we have:

Corollary. *Every Nagata space with 1-continuous semi-metric has a* σ*-closure-preserving* S*-network.*

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On embedding of free abelian topological group $A(X \oplus X)$ into $A(X)$

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$X \oplus X$ denotes the free sum of two copies of X. We consider the following question: for which infinite metrizable compact spaces X the free abelian topological group $A(X \oplus X)$ isomorphically embeds into $A(X)$. While for many natural spaces X such an embedding exists, our main result shows that in general this is not true.

Theorem 1. *Let* M *be a Cook continuum. Then the free abelian topological group* $A(M \oplus M)$ *does not embed into* $A(M)$ *as a topological subgroup.*

Analogous statement is true also for the free boolean group $B(X)$.

$\star$ This is a joint work with Mikołaj Krupski and Sidney Morris.

On some topological properties of the Hausdorff fuzzy metric spaces

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Fuzzy metric, which was first introduced by George and Veeramani in 1994, is one of the most important concepts in the theory of fuzzy topology. In order to explore hyperspaces in given fuzzy metric spaces, Rodriguez-Lopez and Romaguera constructed the Hausdorff fuzzy metric on the family of nonempty compact sets and discussed precompactness, completeness and completion of the Hausdorff fuzzy metric spaces. In the talk, several properties of the Hausdorff fuzzy metric spaces, such as F-boundedness, separability and connectedness are explored.

Minimal base for finite topological space by matrix method

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Topological base plays a foundational role in topology theory. However, few works have been done to nd the base of topological spaces, which would make us difficult to interpret the internal structure of topology spaces. To address this issue, we in this paper study the problem of minimal base algorithm in finite topological spaces based on matrix methods. Firstly, we represent a finite topology space with a Boolean matrix. Then, the properties of minimal base of finite topological spaces are investigated using the matrix. Thirdly, we claim that each minimal base of finite topological spaces can be obtained by the minimal base of subspace employing matrix operations. In the end, a Boolean matrix-based algorithm for finding the minimal base is presented. Experiments are implemented to show the new proposed method is effective for large-scale data.

The k_R-property of free Abelian topological groups and products of sequential fans

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A space X is called a k_R-space, if X is Tychonoff and the necessary and sufficient condition for a real-valued function f on X to be continuous is that the restriction of f to each compact subset is continuous. In this paper, we discuss the k_R-property of products of sequential fans and free Abelian topological groups by applying the κ-fan introduced by Banakh. In particular, we prove the following two results:

(1) The space $S_{\omega_1} \times S_{\omega_1}$ is not a k_R-space.

(2) The space $S_\omega \times S_{\omega_1}$ is a k_R-space if and only if $S_\omega \times S_{\omega_1}$ is a k-space.

These results generalize some well-known results on sequential fans. Furthermore, we generalize some results of Yamada on the free Abelian topological groups by applying the above results. Finally, we pose some open questions about the k_R-spaces.

Fifty years of "Mappings and Spaces"

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The famous survey "Mappings and Spaces" written by A. V. Arhangel'skiĭ in 1966 still gives a powerful driving force to general topology, especially in the theory of generalized metric spaces. This talk provides an overview on its historical significance and practical function for general topology in fifty years, lists some open problems in the survey, and introduces some influence of recent Arhangel'skiĭ's work for the development of general topology in China.

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The ω-resolvability at a point of pseudocompact spaces

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The notion of the resolvability at a point was introduced by E. G. Pytkeev in 1983.

Definition 1. Let κ be any cardinal number. Topological space (X, τ) is called *κ-resolvable at a point* $x \in X$ if X contains κ disjoint spaces A_α such that x is a limit point for all A_α.

Space (X, τ) is called *resolvable at a point* if it is 2-resolvable at this point.

Space (X, τ) is called *maximally resolvable at a point* x if it is $\Delta(x, X)$-resolvable at the x, where $\Delta(x, X) = \min\{|U| : x \in U \in \tau\}$ is a dispersive character of space X at the point x.

It is obvious that a topological space can be resolvable only at a non-isolated point. Pytkeev found a big class of spaces in which non-isolation of a point is a sufficient condition for maximal resolvability at the point.

Examples of irresolvable at any point spaces without isolated points were constructed by A. G. Yel'kin in 1979. Actually, he constructed maximal regular spaces and got the irresolvability at all points like a by-effect.

We establish the following fact.

Theorem 1. *Assume X is a regular topological space and every discrete family of open sets in X is finite. Then X is ω-resolvable at any non-isolated point.*

And as a consequence we establish the following statement.

Theorem 2. *Any pseudocompact topological space is ω-resolvable at every non-isolated point.*

Notes on free topological vector spaces

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We will discuss tightness, k-space property and Fréchet-Urysohn property of subspaces of a free topological vector space.

Definition 1. The free topological vector space $V(X)$ over a Tychonoff space X is a pair consisting of a topological vector space $V(X)$ and a continuous map $i = i_X : X \to V(X)$ such that every continuous mapping f from X to a topological vector space (tvs) E gives rise to a unique continuous linear operator $\overline{f} : V(X) \to E$ with $f = \overline{f} \circ i$.

For a space X and an arbitrary $n \in \mathbb{N}$, we denote by $\mathrm{sp}_n(X)$ the following subset of $V(X)$

$$\mathrm{sp}_n(X) = \{\lambda_1 x_1 + \cdots + \lambda_n x_n : \lambda_i \in [-n, n], x_i \in X, i = 1, \ldots, n\}.$$

Then $V(X) = \bigcup_{n\in N} \mathrm{sp}_n(X)$ and each $\mathrm{sp}_n(X)$ is closed in $V(X)$.

Theorem 1. *The following are equivalent for a metrizable space X:*

(1) $\mathrm{sp}_n(X)$ *is first-countable for each* $n \in N$,

(2) $\mathrm{sp}_n(X)$ *is Fréchet–Urysohn for each* $n \in N$,

(3) $\mathrm{sp}_2(X)$ *is Fréchet–Urysohn,*

(4) X *is compact.*

Theorem 2. *The following are equivalent for space X:*

(1) $V(X)$ *is κ-Fréchet–Urysohn,*

(2) $V(X)$ *is locally compact,*

(3) $V(X)$ *is a q-space,*

(4) X *is finite.*

Theorem 3. *Let X be a Lašnev space, then $V(X)$ is a k-space if and only if $\mathrm{sp}_2(X)$ is a k-space if and only if X is a k_ω-space.*

Theorem 4. *Let X be a Lašnev space, then $V(X)$ is of countable tightness if and only if $\mathrm{sp}_2(X)$ is of countable tightness and if and only if X is separable.*

Problem. Let Y be a closed subset of a metrizable X, is $V(Y)$ homeomorphic to $V(Y, X)$?

2-homeomorphisms and non-nonhomogeneity level

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We will discuss 2-homeomorphisms presented in the paper [A. V. Arhangel'skii, Ju. A. Maksyuta, *Topol. Appl.*, 2018] and how similar technique may be applied to exploring topological space properties.

Definition 1. Topological spaces X and Y are called *2-homeomorphic* if there exist homeomorphic closed subspaces of X and Y such that their complements are also homeomorphic.

Definition 2. A space Y is called *conjugate* to a space X if X is homeomorphic to a closed subspace of Y, and Y is homeomorphic to an open subspace of X.

Theorem 1. *If a space Y is conjugate to a space X, then the spaces X and Y are 2-homeomorphic.*

Example 1. Consider $\mathbb{N}$, the discrete space of natural numbers; and $S = \left\{\frac{1}{n}\right\} \cup \{0\}$, a usual convergent sequence (including the limit point). $\mathbb{N}$ and S are 2-homeomorphic; $\mathbb{N}$ and $\mathbb{N} \times S$ are 2-homeomorphic too, but S and $\mathbb{N} \times S$ are not 2-homeomorphic.

Example 2. The condition in Theorem 1 is sufficient for two spaces to be 2-homeomorphic but is not necessary even in class of compact spaces: the sphere in 3-dimensional Euclidean space and the projective plane are 2-homeomorphic, but no one of them is conjugate to another.

Definition 3. For two spaces X and Y, their *nonsimilarity level* is the number $\mathrm{nsim}(X, Y)$ such that:

1) $\mathrm{nsim}(X, Y) = 0$ if X and Y are homeomorphic;
2) $\mathrm{nsim}(X, Y) \leqslant n$ if there are nonempty homeomorphic open subspaces $X_1 \subset X$ and $Y_1 \subset Y$ and $\mathrm{nsim}(X \setminus X_1, Y \setminus Y_1) \leqslant n - 1$, otherwise $\mathrm{nsim}(X, Y) = \infty$;
3) $\mathrm{nsim}(X, Y) = n$ if $\mathrm{nsim}(X, Y) \leqslant n$ and $\mathrm{nsim}(X, Y) \nleqslant n - 1$.

Definition 4. For a space X, its *nonhomogeneity level* is the number $\mathrm{nhom}(X)$ such that:

1) $\mathrm{nhom}(X) = 0$ if X is homogeneous;
2) $\mathrm{nhom}(X) \leqslant n$ if there is nonempty a homogeneous open subspace $X_1 \subset X$ and $\mathrm{nhom}(X \setminus X_1) \leqslant n-1$, otherwise $\mathrm{nhom}(X) = \infty$;
3) $\mathrm{nhom}(X) = n$ if $\mathrm{nhom}(X) \leqslant n$ and $\mathrm{nhom}(X) \nleqslant n-1$.

Example 3. Let X be the union of a closed circle on Euclidean plane and an isolated point. Then $\mathrm{nhom}(X) = 2$, $\mathrm{nsim}(X, \mathbb{E}^2) = 1$ while $\mathrm{nhom}(\mathbb{E}^2) = 0$.

On factorization properties of function spaces

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For a Tychonoff space X, by $C_p(X)$ we denote the space of all continuous real-valued functions on X, equipped with the topology of pointwise convergence. One of the important questions (due to A. V. Arhangel'skii), stimulating the theory of C_p-spaces for many years and leading to interesting results in this theory, is the problem whether the space $C_p(X)$ is (linearly, uniformly) homeomorphic to its own square $C_p(X) \times C_p(X)$, provided X is an infinite compact or metrizable space. In my talk I will recall several old results and present some recent developments concerning these type of questions. In particular, I will show a metrizable counterexample to this problem for homeomorphisms (a joint result with M. Krupski). I will also show that, for every infinite zero-dimensional Polish space X, the spaces $C_p(X)$ and $C_p(X) \times C_p(X)$ are uniformly homeomorphic (a joint result with R. Górak and M. Krupski).

$\mathbb{R}$-factorizable G-spaces

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Definition. A G-space (G, X, α) is said to be $\mathbb{R}$-*factorizable* in the category G-$\mathcal{T}ych$, if for every continuous real-valued function f on X there exist a separable metrizable G-Tychonoff space (K, Y, β), an equivariant map $(\pi, h)\colon (G, X, \alpha) \to (K, Y, \beta)$ and a real-valued function k on Y such that $f = k \circ h$.

We introduce the notion of a G-space $\mathbb{R}$-factorizable in the category G-$\mathcal{T}ych$ and give its characterization. The $\mathbb{R}$-factorizability of a C-embedded dense subgroup H of a group G is equivalent to the $\mathbb{R}$-factorizability of the G-space (H, G, α), where α is the restriction of the action of G on itself by left translations to $H \times G$. From this it follows that Raikov and Dieudonné completions of an $\mathbb{R}$-factorizable group are $\mathbb{R}$-factorizable in the category G-$\mathcal{T}ych$. We prove that $\mathbb{R}$-factorizability of G-spaces holds in the case of a d-open equivariant image, as a consequence we show that $\mathbb{R}$-factorizability of topological groups is preserved by d-open homomorphisms.

F_σ-mappings between perfectly paracompact spaces

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Definition. A mapping $f\colon X \to Y$ is called an F_σ-*mapping* if f maps F_σ-sets in X to F_σ-sets in Y and f^{-1} maps F_σ-sets in Y to F_σ-sets in X.

A detailed study of F_σ-mappings between absolute Suslin metric spaces was initiated by J. E. Jayne, C. A. Rogers and R. W. Hansell. In particular, they showed that an F_σ-mapping is in fact piecewise closed under some assumptions. Such investigations were continued by P. Holický and J. Spurný.

We will discuss how those results can be generalized to Suslin F_σ-subsets of perfectly paracompact spaces. We will also analyse the relationship between F_σ-mappings and F_σ-measurable mappings.

Some aspects of dimension theory for topological groups

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We discuss dimension theory in the class of all topological groups. For locally compact topological groups there are many classical results in the literature. Dimension theory for non-locally compact topological groups is mysterious. It is for example unknown whether every connected (hence at least 1-dimensional) Polish group contains a homeomorphic copy of $[0, 1]$. And it is unknown whether there is a homogeneous metrizable compact space the homeomorphism group of which is 2-dimensional. Other classical open problems are the following ones. Let G be a topological group with a countable network. Does it follow that $\dim G = \operatorname{ind} G = \operatorname{Ind} G$? The same question if X is a compact coset space. We also do not know whether the inequality $\dim(G \times H) \leq \dim G + \dim H$ holds for arbitrary topological groups G and H which are subgroups of σ-compact topological groups. The aim of this talk is to discuss such and related problems.

⋆ This is a joint work with A. V. Arhangel'skii.

The local density and the local weak density of superextension and N_τ^φ-kernel of a topological space

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We say that a topological space X is *locally τ-dense* at a point $x \in X$ if τ is the smallest cardinal number such that x has a τ-dense neighborhood in X. The local density at a point x is denoted by $ld(x)$.

The *local density* of a space X is defined as the supremum of all numbers $ld(x)$ for $x \in X$; this cardinal number is denoted by $ld(X)$.

A topological space is *locally weakly τ-dense* at a point $x \in X$ if τ is the smallest cardinal number such that x has a neighborhood of weak density τ in X. The weak density at a point x is denoted by $lwd(x)$.

A topological space X is called *locally weakly τ-dense* if it is weakly τ-dense at each point $x \in X$.

The *local weak density* of a topological space X is defined in the following way:

$$lwd\,(X) = \sup\,\{lwd\,(x) : x \in X\}\,.$$

A system $\xi = \{F_\alpha :\ \alpha \in A\}$ of closed subsets of a space X is called *linked* if any two elements from ξ intersect [1].

A. V. Ivanov defined the space NX of complete linked systems (CLS) of a space X in the following way:

Definition 1. A linked system μ of closed subsets of a compact X is called a *complete linked system* (a CLS) if, for any closed set of X, the condition

“Any neighborhood OF of the set F consists of a set $\Phi \in \mu$”

implies $F \in \mu$ [2].

The set NX of all complete linked systems of a compactum X is called the *space NX of CLS of X*. This space is equipped with the topology, the open base of which consists of all sets of the form

$$\begin{aligned} E &= O(U_1, U_2, \ldots, U_n)\langle V_1, V_2, \ldots, V_s\rangle \\ &= \{\mu \in NX : \text{for any } i = 1, 2, \ldots, n \text{ there exists } F_i \in \mu \\ &\qquad \text{such that } F_i \subset U_i, \text{ and } F \cap V_j \neq \varnothing \text{ for any } j = 1, 2, \ldots, s,\ F \in \mu\}, \end{aligned}$$

where $U_1, U_2, \ldots, U_n$, $V_1, V_2, \ldots, V_s$ are nonempty open in X sets [2].

Definition 2. Let X be a compact space, φ be a cardinal function and τ be an arbitrary cardinal number. We call the *N_τ^φ-kernel* of a topological space X the space

$$N_\tau^\varphi X = \{\mu \in NX : \exists F \in \mu : \varphi(F) \leq \tau\}.$$

Theorem 1. *Let X be an infinity compact space and $\varphi = d, \tau = \aleph_0$. Then:*

1) *$ld(N_\tau^\varphi X) \neq ld(\lambda X)$;*

2) *$lwd(N_\tau^\varphi X) \neq lwd(\lambda X)$.*

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The spaces $C_\lambda(X)$: decomposition into a countable union of bounded subspaces

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For a Tychonoff space X and a family λ of subsets of X, we denote by $C_\lambda(X)$ the space of all real-valued continuous functions on X with the set-open topology. In particular, if λ consists of all finite subsets of X then $C_\lambda(X) = C_p(X)$.

A space X is said to be *Menger* [2] if for every sequence $\{\mathcal{U}_n : n \in \omega\}$ of open covers of X, there are finite subfamilies $\mathcal{V}_n \subset \mathcal{U}_n$ such that $\bigcup\{\mathcal{V}_n : n \in \omega\}$ is a cover of X.

In [1], A. V. Arhangel'skii proved that $C_p(X)$ is Menger, if and only if X is finite. We will discuss how one can generalize this result for $C_\lambda(X)$.

Theorem. *For a Tychonoff space X and a π-network λ of X, the following statements are equivalent:*

1) $C_\lambda(X)$ *is σ-compact;*
2) $C_\lambda(X)$ *is Alster;*
3) $C_\lambda(X)$ *is projectively σ-compact and Lindelöf;*
4) $C_\lambda(X)$ *is Hurewicz;*
5) $C_\lambda(X)$ *is Menger;*
6) X *is a pseudocompact, $D(X)$ is a dense C^*-embedded set in X and the family λ consists of all finite subsets of $D(X)$, where $D(X)$ is the set of isolated points of X.*

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Roughness on topological vector spaces

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We will define rough vector spaces and topological rough vector spaces. Then we investigate properties of these topological algebraic structures, and also investigate rough convexity.

The property of having a Luzin π-base is not preserved by products

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Recall that a *Luzin scheme* on a set X is a family $\langle L_s \rangle_{s \in {}^{<\omega}\omega}$ of subsets of X such that

➢ $L_s \supseteq L_{s^\frown n}$ for all $s \in {}^{<\omega}\omega$, $n \in \omega$;

➢ $L_{s^\frown n} \cap L_{s^\frown m} = \varnothing$ for all $s \in {}^{<\omega}\omega$ and $n \neq m \in \omega$,

where ${}^{<\omega}\omega$ is the set of finite sequences of natural numbers and

$$\langle s_0, \ldots, s_{k-1} \rangle^\frown n := \langle s_0, \ldots, s_{k-1}, n \rangle.$$

A Luzin scheme on X is *strict* iff

➢ $L_{\langle\rangle} = X$;

➢ $L_s = \bigcup_{n \in \omega} L_{s^\frown n}$ for all $s \in {}^{<\omega}\omega$;

➢ $\bigcap_{n \in \omega} L_{a \restriction n}$ is a singleton for all $a \in {}^{\omega}\omega$,

where $\langle\rangle$ is the empty sequence, ${}^{\omega}\omega$ is the set of infinite sequences of natural numbers, and $a \restriction n$ is the restriction of a to its first n arguments. A Luzin scheme on X is *open* iff each L_s is an open subset of X.

Definition. A *Luzin π-base* for a space X is an open strict Luzin scheme on X such that for any point $p \in X$ and any its neighbourhood $O(p)$, there are $s \in {}^{<\omega}\omega$ and $k \in \omega$ so that $p \in L_s$ and $\bigcup_{n \geqslant k} L_{s^\frown n} \subseteq O(p)$.

The Baire space ω^ω and the Sorgenfrey line $\mathcal{S}$ have a Luzin π-base [2]. All at most countable powers of $\mathcal{S}$ and all at most countable powers of the irrational Sorgenfrey line have a Luzin π-base [4]. If a space X has a Luzin π-base, then the products $X \times \omega^\omega$, $X \times \mathcal{S}$, and $X \times \mathcal{S}^\omega$ have a Luzin π-base [4], and also $X \setminus F$ has a Luzin π-base whenever F is σ-compact [3] (but even a dense open subset of X can be without a Luzin π-base).

If a space X has a Luzin π-base, then it can be mapped onto the Baire space ω^ω by a continuous one-to-one map [2] and also X can be mapped onto ω^ω by a continuous open map [2] (hence X can be mapped by a continuous open map onto an arbitrary Polish space, see [1]). If a space X has a Luzin π-base, then it has a countable π-base and a countable pseudobase (both with clopen members); and also X is a Choquet space (but it can be not strong Choquet even in separable metrizable case). For each $A \subseteq \omega^\omega$, there exists a separable metrizable space with a Luzin π-base that contains a closed subspace homeomorphic to A.

Theorem. *There exist two spaces with Luzin π-bases whose product has no Luzin π-base.*

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q-equivalent not t-equivalent spaces

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Spaces X and Y are called q-equivalent if there is a bijection $f\colon C_p(X) \to C_p(Y)$ such that both maps f and f^{-1} are continuous when restricted to compact subspaces of $C_p(X)$ and $C_p(Y)$ respectively.

We present examples of spaces which are q-equivalent but not t-equivalent, thus answering a question of A. V. Arhangelskii.

Subgroups of products of certain paratopological (semitopological) groups

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In the first part of this report, we give some sufficient conditions under which a paratopological group is topologically isomorphic to a subgroup of a product of strongly metrizable paratopological groups. In the second part of this report, we show that a regular (Hausdorff, T_1) semitopological group G admits a homeomorphic embedding as a subgroup into a product of regular (Hausdorff, T_1) first-countable semitopological groups which are σ-spaces if and only if G is locally ω-good, ω-balanced, $\mathrm{Ir}(G) \leq \omega$ ($\mathrm{Hs}(G) \leq \omega$, $\mathrm{Sm}(G) \leq \omega$) and with the property that for every open neighborhood U of the identity e of G the cover $\{xU : x \in G\}$ has a basic refinement $\mathcal{F}$ which is σ-discrete with respect to a countable family $\mathcal{V}$ of open neighborhoods of e. In the last part of this report, we give an internal characterization of projectively T_i second-countable semitopological groups, for $i = 0, 1, 2$.

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Variational principles in optimization and topological games

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In this talk we will present relations between the validity of certain variational principles in optimization and existence of winning strategies in some topological games played in the underlying space. A typical variational principle in optimization has the following setting: given a bounded from below (lower semi-) continuous proper function $f\colon X \to \mathbb{R}$ defined in a completely regular topological space X, we are looking for conditions under which we can perturb f by enough rich ("rich" means big from topological point of view) set of continuous bounded functions g defined in X such that the perturbation $f+g$ attains its minimum in X (or even stronger, is well-posed). It has turned out that, in order to obtain such kind of principles, the space X must admit a winning strategy for one of the players in certain type of topological games played in X. Relations between the validity of such variational principles and other fundamental properties of topological spaces, like fragmentability will be presented as well.

$\star$ This is a joint work with M. M. Choban and P. S. Kenderov.

Some cardinal properties of functors of finite degree

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In the paper, hereditary cardinal-valued properties of normal functors in the category of compact spaces are investigated.

Theorem 1. *Let X be an infinite compactum such that $C_p(X)$ is a Lindelöf Σ-space, and $F_n\colon \mathcal{C}omp \to \mathcal{C}omp$ be a normal functor of degree n. Then the following cardinal functions* (for their definitions, see [2]) *are preserved by F_n applied to X:*

1) *hereditary cellularity, $hc(F_n(X)) = hc(X)$;*
2) *hereditary density, $hd(F_n(X)) = hd(X)$;*
3) *hereditary π-weight, $h\pi w(F_n(X)) = h\pi w(X)$;*
4) *hereditary Shanin number, $hsh(F_n(X)) = hsh(X)$;*
5) *spread, $s(F_n(X)) = s(X)$.*

Recall that *Corson compacta* [1, 2] are compact subsets of a Σ-product of separable metrizable spaces (or, what is the same, compact subsets of the Σ-product of segments).

Theorem 2. *Let X be an infinite Corson compactum such that $C_p(C_p(X))$ is a Lindelöf Σ-space, and $F_n\colon \mathcal{C}omp \to \mathcal{C}omp$ be a normal functor. Then F_n being applied to X preserves the same five cardinal functions.*

A topological space X is called a *Dante space* [3], if for any infinite cardinal number τ there exists a dense subspace $X' \subset X$ which is simultaneously τ-monolithic in itself and τ-suppressed by the space X.

Theorem 3. *For any infinite Dante compact space X, an arbitrary normal functor $F_n\colon \mathcal{C}omp \to \mathcal{C}omp$ applied to X preserves the cardinal functions listed in Theorem 1, and also character and tightness.*

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The Menger property of $C_p(X, 2)$ and related matters

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A space X is said to be *Menger* if for any sequence $\{\mathcal{U}_n : n \in \omega\}$ of open covers of X, there exist finite $\mathcal{V}_n \subset \mathcal{U}_n$ $(n \in \omega)$ such that $\bigcup\{\mathcal{V}_n : n \in \omega\}$ is a cover of X. A. V. Arhangel'skiĭ proved that $C_p(X)$ is Menger if and only if X is finite, in "Hurewicz spaces, analytic sets and fan tightness of function spaces" [*Soviet Math. Doklady*, **33** (1986), 396–399]. For a zero-dimensional space X, let $C_p(X, 2)$ be the space of all $\{0, 1\}$-valued continuous functions with the topology of pointwise convergence. Even if $C_p(X, 2)$ is Menger, X need not be finite. Indeed, if a space X is discrete, then $C_p(X, 2)$ is compact (so, Menger). Bernal-Santos and Tamariz-Mascarúa gave several results when $C_p(X, 2)$ is Menger, in "The Menger property on $C_p(X, 2)$" [*Topol. Appl.*, **183** (2015), 110–126]. In this talk, we give some improvements of them.

Quotients of topological groups

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We show that there exists a closed subgroup H of a topological group G such that G/H is a first-countable compact space, but G/H is not submetrizable. On the other hand, if H is a closed neutral subgroup of a topological group G such that G/H is first-countable, then G/H is metrizable. This result is a generalization of the Birkhoff–Kakutani theorem. Also, if H is a closed neutral subgroup of a topological group G such that G/H has countable pseudocharacter, then G/H is submetrizable. We study under which conditions the quotient space G/H is submetrizable, where H is a closed subgroup of a paratopological group G.

⋆ This is a joint work with Manuel Fernández and Mikhail Tkachenko.

Idempotent ultrafilters are not selective

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We show that in algebras with sufficiently cancellative operations, their Čech–Stone compactifications cannot have idempotent ultrafilters that are selective. This generalizes previous observations by Hindman and Protasov on groups and countably complete ultrafilters to wider classes of algebras and ultrafilters.

Inner points and exact Milyutin maps

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The famous convex-valued Michael’s theorem on selections states that an LSC mapping F with a paracompact domain X, Banach (Fréchet) range space Y and with non-empty convex closed values admits a single-valued continuous selection f. Much less well-known theorem (Theorem $3.1'''$) of Michael states that the closedness of values $F(x)$ can sometimes be weakened with simultaneous strengthening of information on a selection. Namely, for a completely normal X, separable Y, a single-valued continuous selection exists whenever each value $F(x)$ contains all inner points of its own closure.

As for the notion of *inner point* x of a convex set C, there are various (in the infinite-dimensional case) approaches, but here it means that x does not belong to any face (= to a proper closed convex extreme set) of C.

Lemma. *For the space of all probability measures $P(M)$ on a Polish space M endowed with the weak convergency topology, the subset*

$$P_{\text{exact}}(M) = \{\mu \in P(M) : \operatorname{supp}(\mu) = M\}$$

of exact measures contains all inner points of $P(M) = \operatorname{clos}(P_{\text{exact}}(M))$.

So, by applying the above selection theorem one can obtain

Proposition. *For each LSC mapping G with a completely normal domain X, Polish range space M and with non-empty closed values, there is a continuous mapping $E\colon X \to P(M)$ such that for every $x \in X$ the probability measure $E(x)$ is exact on $G(x)$, $\operatorname{supp}(E(x)) = G(x)$.*

Earlier, the statement was proved for the case $G = g^{-1}$ where g is an open surjection between Polish spaces.

Weak α-favourability in topological spaces and groups

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For a topological space X, the *Banach–Mazur game* on X is played between two players. At round 1, Player A selects a non-empty open subset A_1 of X, and Player B responds with choosing a non-empty open subset B_1 inside of A_1. At round 2, Player A selects a non-empty open subset $A_2 \subseteq B_1$, and Player B responds by choosing a non-empty open subset $B_2 \subseteq A_2$. The game continues to infinity producing a decreasing sequence $A_1 \supseteq B_1 \supseteq A_2 \supseteq B_2 \supseteq \dots$ of non-empty open subsets of X. Player B wins if $\bigcap_{n\in\mathbb{N}} A_n = \bigcap_{n\in\mathbb{N}} B_n \neq \emptyset$; otherwise Player A wins.

A topological space X is called *weakly α-favourable* if Player B has a winning strategy in the Banach–Mazur game on X.

We prove the following factorization theorem for weak α-favourability.

Theorem 1. *Let $h\colon X \to Z$ be a continuous map from a Tychonoff weakly α-favourable space X to a separable metric space Z. Then there exist a weakly α-favourable separable metric space Y and two continuous maps $g\colon X \to Y$, $f\colon Y \to Z$ such that $h = f \circ g$ and $Y = g(X)$.*

Since a weakly α-favourable metric space contains a dense completely metrizable subspace, this theorem implies similar factorization theorems for many other completeness-type properties (like Oxtoby completeness and Todd completeness, for example).

Definition 2. We say that a topological group X is *Polish factorizable* provided that for every continuous homomorphism $h\colon X \to Z$ from X to a separable metric group Z, there exist a Polish group Y and continuous homomorphisms $g\colon X \to Y$, $f\colon Y \to Z$ such that $h = f \circ g$ and $Y = g(X)$.

Recall that a topological group G is called *ω-bounded* (*precompact*) provided that for every open neighbourhood U of the identity of G one can find an at most countable (respectively, finite) set S such that $G = SU$.

Theorem 3. *An ω-bounded weakly α-favourable Hausdorff group is Polish factorizable.*

From this result we obtain a new characterization of pseudocompactness in groups.

Corollary 4. *A Hausdorff topological group is pseudocompact if and only if it is both precompact and weakly α-favourable.*

This corollary implies an earlier result by the authors that a weakly pseudocompact precompact Hausdorff group is pseudocompact, and gives a strong positive answer to Problem 7.5 from [S. García-Ferreira, R. Rojas-Hernández, Á. Tamariz-Mascarúa, "Completeness type properties on $C_p(X, Y)$ spaces," *Topol. Appl.*, **219** (2017), 90–110].

Theorem 5. *Let G be a dense subgroup in the product $H = \prod_{i\in I} H_i$ of separable metric groups H_i. Then the following conditions are equivalent:*

(1) *G is weakly α-favourable;*

(2) *G is Telgársky complete;*

(3) *G is strongly Oxtoby complete;*

(4) *all groups H_i are Polishable and $\pi_J(G) = \prod_{i\in J} H_i$ for every at most countable subset J of I, where $\pi_J : H \to \prod_{i\in J} H_i$ is the projection.*

Our results give a positive answer to Problem 7.6 from the paper cited above for the class of separable metric groups and precompact groups.

⋆ This is a joint work with Alejandro Dorantes-Aldama (Mexico).

Duality in topological and convergence groups

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Pontryagin's duality theorem is one of the theorems of analysis which makes sense beyond the assumption of local compactness, and the subject is developing as a tool for the structural dismemberment of topological and convergence groups which are not necessarily locally compact. We make an attempt to present the state of the art in the Pontryagin duality theory of abelian groups with limit related structures and to point out the research issue in this growing field. Throughout, an attempt is made to present the influence of functional analysis on the development of the subject.

On homeomorphisms of zero-dimensional compacta

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The problem considered concerns extensions of homeomorphisms of 0-dimensional compacta. The following theorem is proved:

Theorem. *Let $D^{\aleph_0}$ be the Cantor discontinuum, Y and Y' be a pair of homeomorphic to each other closed subsets of $D^{\aleph_0}$. A homeomorphism $h\colon Y \to Y'$ can be extended to an autohomeomorphism $h'\colon D^{\aleph_0} \to D^{\aleph_0}$ if and only if for any $y \in Y$ the following two conditions are equivalent:*

1) *the point y belongs to interior of Y;*

2) *the point $h(y)$ belongs to the interior of Y'.*

As a corollary, one obtains that every homeomorphism between closed, nowhere dense subsets of the Cantor discontinuum can be extended over the whole discontinuum.

Arrow ultrafilters and topological groups

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A relationship between the existence of κ-arrow ultrafilters and topological groups with certain properties is discussed. New properties of κ-arrow ultrafilters are determined.

Cardinal invariants for the G_δ topology

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Given a topological space X, we indicate with X_δ the topology on the underlying set of X which is generated by the G_δ subsets of X. We will survey joint work with Bella and Szeptycki regarding cardinal invariants for the G_δ topology. In particular, we will give bounds for the Lindelöf number, the weak Lindelöf number and the spread of X_δ in terms of their value on X, and construct examples to show the sharpness of our bounds, one of which solves a 1969 question due to A. V. Arhangel'skii. We will finally show how to apply our bounds to obtain cardinal estimates for homogeneous compacta.

The Rudin–Keisler ordering of P-points under $\mathfrak{b} = \mathfrak{c}$

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M. E. Rudin proved under CH that for each P-point there exists another P-point strictly RK-greater. This result was proved under $\mathfrak{p} = \mathfrak{c}$ by A. Blass, who also showed that each RK-increasing ω-sequence of P-points is upper bounded by a P-point, and that there is an order embedding of the real line into the class of P-points with respect to the RK-order. The results cited above are proved here under a (weaker) assumption $\mathfrak{b} = \mathfrak{c}$.

A. Blass asked in 1973 which ordinals can be embedded in the set of P-points, and pointed out that such an ordinal cannot be greater than $\mathfrak{c}^+$. This question is answered by showing (under $\mathfrak{b} = \mathfrak{c}$) that there is an order embedding of $\mathfrak{c}^+$ into P-points.

The techniques of the proofs in this paper are based on a method of contours, which enables one to argue more easily and concisely.

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On space of continuous functions given on certain modifications of linearly ordered spaces

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We consider $C_p(X)$, where X is a modification of the Sorgenfrey line or an Hattori space [Y. Hattori, *Mem. Fac. Sci. Eng. Shimane Univ. Ser. B Math. Sci.*, 2010]. For a subset A of the real line $\mathbb{R}$, the modification of the Sorgenfrey line, denoted S_A, is defined as follows: a basis of neighbourhoods for $x \in A$ is given by the right open intervals $[x, y)$, $x < y$, $y \in \mathbb{R}$, and a basis for $x \notin A$ is given by the left open intervals $(y, x]$, $x > y$, $y \in \mathbb{R}$. For a subset $A \subset \mathbb{R}$, the Hattori space, denoted $H(A)$, is defined as follows: a basis of neighbourhoods for $x \in A$ is given by the usual Euclidean neighbourhoods of x and a basis for $x \notin A$ is given by the left open intervals $(y, x]$, $x > y$, $y \in \mathbb{R}$. Notice that $S_\emptyset = H(\emptyset) = \mathbb{S}$ is the Sorgenfrey line. We establish two new facts.

Theorem 1. *Let $A \subset \mathbb{R}$. Then the following conditions are equivalent:*

(i) *the spaces $\mathbb{S}$ and S_A are homomorphic;*

(ii) *A is an F_σ and G_δ-absolute space;*

(iii) *the spaces $C_p(\mathbb{S})$ and $C_p(S_A)$ are linear homomorphic.*

Theorem 2. *Let $A \subset \mathbb{R}$. Then the following conditions are equivalent:*

(i) *the spaces $\mathbb{S}$ and $H(A)$ are homomorphic;*

(ii) *A is scattered;*

(iii) *the spaces $C_p(\mathbb{S})$ and $C_p(H(A))$ are linear homomorphic.*

Some new classes of ideals of $C(X)$ and λX

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In this talk first we give a new representation for closed ideals in $C(X)$ and the intersections of maximal ideals in $C^*(X)$. Next, for a completely regular Hausdorff space X, we construct a space λX in βX containing υX and show that X is Lindelöf if and only if X coincides with λX. We also characterize the spaces X for which some familiar ideals of $C(X)$ would be closed ideals. For instance, it is shown that $C_\infty(X)$ is the intersection of all free maximal ideals of $C(X)$ if and only if every open locally compact σ-compact subset of X is relatively pseudocompact.

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C_p-theory applied to Model Theory

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I will first briefly survey the many areas of my research that have been strongly influenced by Prof. Arhangel'skiĭ's work. Then I will talk about how C_p-theory (an entirely new area for me) is applied to Model Theory, that branch of Mathematical Logic that deals with semantics (rather than syntax) and, in particular, definability. In particular, in collaboration with model theorists J. Iovino, X. Caicedo, and C. Eagle, we extend the solution by Casazza and Iovino of Gowers' question concerning the definability of the Tsirelson Banach space beyond first-order logic.

⋆ This is a joint work with Jose Iovino.

Gaps in lattices of topological group topologies

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The family of all topological group topologies, with the operations of taking the meet and join of topologies, forms a complete lattice. One of the many intriguing questions about this kind of lattices is the existence of gaps. We focus our attention on Hausdorff predecessors of locally compact topological group topologies and provide an almost complete description of these predecessors. An interesting relation between these predecessors and the Bohr topology of a locally compact abelian group is established as well.

We show, for example, that every Hausdorff predecessor, σ, of a (noncompact) locally compact topological group topology τ on an abelian group G contains the *Bohr* topology of the group (G, τ) or, equivalently, every continuous character of the group (G, τ) is σ-continuous. Furthermore, we show that the group (G, σ) is not metrizable, while it is consistent with ZFC that the cardinality of any local base at the identity of (G, σ) is at least 2^ω.

Several open problems on the gaps in the lattices of topological group topologies on abelian groups will be posed in the lecture.

⋆ This is a joint work with Wei He, Dekui Peng, and Zhiqiang Xiao.

Some remarks about Alexandroff's hypothesis concerning V^p-continua

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Let X be a metric space. The n-dimensional diameter $d_n(X)$ of X is the number $\inf\{\mathrm{mesh}(\mathcal{U})\}$, where $\mathcal{U}$ runs over the set of all open coverings of X with $\mathrm{ord}(\mathcal{U}) \leq n+1$. We say that a metric space X is an (n, ε)-connected between its disjoint closed subsets A and B if for every partition C between A and B we have $d_{n-2}C \geq \varepsilon$.

Recall next, that a metric compact space X is a V^p-continuum (P. S. Alexandroff, 1956) if $\dim X = p$ and for every disjoint pair F and G of closed subsets of X with non-empty interiors there exists $\varepsilon > 0$ such that X is (p, ε)-connected between F and G.

In 1956 P. S. Alexandroff introduced a hypothesis providing a condition under which the sum of two V^p-continua is a V^p-continuum. Later (1979) it was proven by myself that the Alexandroff's hypothesis is not valid.

In the present talk we discuss some development concerning that hypothesis of P. S. Alexadroff.

Some results on paracompact remainders

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All spaces under discussion are Tychonoff. A remainder of a space X is the subspace $bX \setminus X$ of a compactification bX of X. We investigate how some paracompactness type properties of a space interact with properties of remainders of this space.

Spectral representations of topological groups and near-openly generated groups

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The class of of near-openly generated topological groups is introduced and investigated. It is a topological subclass of $\mathbb{R}$-factorizable groups. We provide both topological and external characterizations of this class using spectral methods.

⋆ This is a joint work with K. Kozlov

Constructing a minimal left ideal of $(\omega^*, +)$ which is a weak P-set

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We modify Kunen's construction of weak P-points to construct a minimal left ideal of the semigroup $(\omega^*, +)$ which is also a weak P-set.

⋆ This is a joint work with Will Brian.

On semiconic idempotent commutative residuated lattices

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In this paper, we study semiconic idempotent commutative residuated lattices. An algebra of this kind is a *semiconic generalized Sugihara monoid* (SGSM) if it is generated by the lower bounds of the monid identity. We establish a category equivalence between SGSMs and Brouwerian algebras with a *strong nucleus*. As an application, we show that central semiconic generalized Sugihara monoids are strongly amalgamable.

Capturing topological spaces by countable elementary submodels

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Given a Tychonoff space (X, τ) and an elementary submodel M of a sufficiently large initial fragment of the universe such that $(X, \tau) \in M$, we can define a quotient-like topological space X/M by a method introduced by Bandlow and Dow. The space X/M is always separable metrizable if M is countable. We give sufficient conditions for X such that X/M is homeomorphic to some familiar subspaces of the real line (2^ω, ω^ω, intervals, etc.). To this end, we establish several preservation theorems in the form that "if X has $\Phi + \mathcal{P}$, then X/M has $\mathcal{P}$ for any countable M." Questions are posed.

Тезисы докладов

О функторах полуаддитивных σ-гладких функционалов

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В работе вводится функтор OS_σ полуаддитивных σ-гладких функционалов в категорию тихоновских пространств $\mathcal{T}ych$, который продолжит функтор $OS\colon \mathcal{C}omp \to \mathcal{C}omp$ полуаддитивных функционалов. Доказывается, что функтор $OS_\sigma\colon \mathcal{T}ych \to \mathcal{T}ych$ переводит Z-вложения во вложения. А также доказывается, что пространство $OS_\sigma(X)$ замкнуто лежит в пространстве $O_\sigma(X)$ слабо аддитивных σ-гладких функционалов, полно по Хьюитту для любого тихоновского пространства X.

О топологических группах

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1. В 1954 году Jennings ввел в широкое изучение группу $J(\Bbbk)$ формальных степенных рядов $f(x) = x + \alpha_1 x^2 + \alpha_2 x^3 + \dots = x(1 + \alpha_1 x + \alpha_2 x^2 + \dots)$ с коэффициентами в коммутативном кольце $\alpha_n \in \Bbbk$, — в качестве операции рассматривается композиция. В случае поля $\Bbbk = \mathbb{Z}_p$ группа $J(\mathbb{Z}_p)$ обладает свойством универсальности по вложению счётных p-групп и называется Ноттингемской группой. Для колец $\mathbb{Z}$, $\mathbb{Z}_2$ и $\mathbb{Z}_p$, $p \geq 3$ (топологические) коммутаторы групп Дженнингса существенно отличаются [1].

В докладе рассматривается многомерный аналог группы Дженнингса и показывается, что задача описания (топологического) коммутатора приводит к ответу несколько иному, чем в одномерном случае.

2. Обсуждается задача вычисления числа топологизаций абстрактной абелевой группы G бесконечной мощности $\mathfrak{m}$. Основной наш результат здесь был

получен в совместной работе [2]. К сожалению, в момент публикации авторы не знали о соответствующих исследованиях, которые провели W. W. Comfort и Dieter Remus в статьях [3–5]. Считаем необходимым отметить эту недоработку в докладе.

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Паранормальность подмножеств пространств функций

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Топологическое пространство называется *паранормальным* [P. Nyikos, *Topol. Proc.,* **9**(2) (1984), p. 367], если для любой счётной дискретной системы замкнутых множеств $\{D_n : n < \omega\}$ найдётся локально конечная система открытых множеств $\{U_n : n < \omega\}$ такая, что для всех $n < \omega$ выполняется $D_n \subset U_n$ и $D_m \cap U_n \neq \emptyset$ в том и только в том случае, когда $D_n = D_m$. Все нормальные пространства и все счётно паракомпактные пространства являются паранормальными.

Известно [А. В. Архангельский, «Топологические пространства функций», *Изд-во МГУ*, Москва, 1989, теорема 1.5.6], что *пространство функций $C_p(X)$ наследственно нормально в том и только в том случае, если оно совершенно нормально.* Предлагается следующее обобщение этой теоремы.

Теорема 1. *Пространство $C_p(X)$ наследственно паранормально в том и только в том случае, если оно совершенно нормально.*

Непосредственным следствием теоремы 1 является утверждение о том, что если любое подмножество пространства $C_p(X)$ счётно паракомпактно, то $C_p(X)$ совершенно нормально.

Теорема 2. *Любое F_σ-подмножество пространства $C_p(X)$ паранормально в том и только в том случае, если $C_p(X)$ нормально.*

Равномерно перистые пространства

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Рассматриваются равномерные аналоги перистых, полных по Чеху и близких к ним пространств.

Равномерное пространство $(X,\mathcal{U})$ называется *равномерно τ-перистым*, где $\aleph_0 \le \tau \le w(x)$, если существует псевдоравномерность $\vartheta \subset \mathcal{U}$, удовлетворяющая следующим условиям:

1) $\omega(\vartheta) \le \tau$,
2) $\bigcap\{\alpha(x) : \alpha \in \vartheta\} = K_x$ компактно для любого $x \in X$,
3) система $\{\alpha(K_x) : \alpha \in \vartheta\}$ является фундаментальной системой окрестностей компакта K_x для каждого $x \in X$.

Равномерно $\aleph_0$-перистые пространства называются просто *равномерно перистыми.*

Всякое метризуемое равномерное пространство является равномерно перистым.

Теорема. *Для равномерного пространства $(X,\mathcal{U})$ следующие условия равносильны:*

1) *равномерное пространство $(X,\mathcal{U})$ является равномерно τ-перистым;*
2) *равномерное пространство $(X,\mathcal{U})$ отображается на некоторое равномерное пространство (Y,ϑ) веса $\le \tau$ посредством совершенного равномерно непрерывного отображения.*

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Топологическая классификация пространств бэровских конечнозначных функций

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Полная линейная топологическая классификация банаховых пространств $C[0,\alpha]$ всех непрерывных функций на компактных отрезках ординалов была проведена в работах Бессаги и Пелчинского, Семадени, Гулько и Оськина и Кислякова. Позже появилась линейная топологическая классификация этих же пространств с топологией поточечной сходимости (Баарс и де Грот, Гулько). Затем авторами была проведена линейная топологическая классификация пространств бэровских функций $B_p([0,\alpha],Y)$, где Y — это либо вещественная прямая, либо группа $\mathbb{Z}_2$, а также пространств $C_p([0,\alpha],\mathbb{Z}_p)$. Недавно авторам удалось доказать, что топологическая классификация пространств $B_p[0,\alpha]$ совпадает с линейной топологической классификацией этих пространств.

Здесь мы проводим топологическую классификацию пространств бэровских функций $B_p([0,\alpha],Z_2)$. Получена следующая

Теорема. *Пусть α и β — бесконечные ординалы и $\alpha \le \beta$. Тогда пространства $B_p([1,\alpha],Z_2)$ и $B_p([1,\beta],Z_2)$, гомеоморфны тогда и только тогда, когда выполняется одно из следующих взаимоисключающих условий:*

1) $\omega \le \alpha \le \beta < \omega_1$;
2) $\omega_1 \le \alpha \le \beta < \omega_2$;
3) $\tau\cdot n \le \alpha \le \beta < \tau\cdot(n+1)$, *где* $\tau \ge \omega_2$ *— начальный регулярный ординал и* $n < \omega$;
4) $\tau \cdot \sigma \le \alpha \le \beta < \tau \cdot \sigma^+$, *где* $\tau \ge \omega_2$ *— начальный регулярный ординал,* σ *— такой начальный ординал, что* $\omega \le \sigma < \tau$ *и* σ^+ *— наименьший начальный ординал, больший, чем* σ;

5) $\tau^2 \leq \alpha \leq \beta < \tau^+$, *где* $\tau \geq \omega_2$ — *начальный регулярный ординал и* τ^+ — *наименьший начальный ординал, больший, чем* τ;

6) $\tau \leq \alpha \leq \beta < \tau^+$, *где* $\tau \geq \omega_2$ — *начальный сингулярный ординал.*

О суперпаракомпактности отображений вида λf

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Настоящая работа является продолжением работы [1] в области исследований теории суперпаракомпактных пространств.

Система ω подмножеств множества X называется *звёздно счётной* (*конечной*), если каждый элемент системы ω пересекается не более чем со счётным (конечным) числом элементов этой системы. Конечная последовательность подмножеств $M_0, \ldots, M_s$ множества X называется *цепью, связывающей множества* M_0 *и* M_s, если $M_{i-1} \cap M_i \neq \emptyset$ при любом $i = 1, \ldots, s$. Система ω подмножеств множества X называется *сцепленной*, если для любых множеств M и M' этой системы существует такая цепь элементов системы ω, что первый элемент цепи есть множество M, а последний — множество M'.

Максимальные сцепленные подсистемы системы ω называются *компонентами сцепленности* (или компонентами) системы ω. При этом компоненты звёздно счётной системы ω счётны и тела различных компонент системы ω дизъюнктны [2].

Для пространства X и системы его подмножеств $\omega = \{O_\alpha : \alpha \in A\}$ полагаем $[\omega] = [\omega]_X = \{[O_\alpha]_X : \alpha \in A\}$.

Звёздно конечное открытое покрытие пространства X называется [2] *конечнокомпонентным*, если все его компоненты сцепленности конечны.

Для тихоновского пространства X через βX обозначим его Стоун-Чеховскую компактификацию.

Напомним, что непрерывное отображение $f\colon X \to Y$ называется [2] T_0-*отображением*, если для каждой пары различных точек x, x' таких, что $f(x) = f(x')$, хотя бы у одной из этих точек в X найдётся окрестность, не содержащая другую точку. Непрерывное отображение $f\colon X \to Y$ называется [2] *вполне регулярным*, если для любой точки $x \in X$ и любого замкнутого в X множества F, не содержащего точку x, найдётся такая окрестность O

точки $f(x)$, что в трубке $f^{-1}O$ множества $\{x\}$ и F функционально отделимы. Наконец, вполне регулярное T_0-отображение называется *тихоновским отображением.*

Непрерывное замкнутое отображение $f\colon X \to Y$ называется *бикомпактным* (И. А. Вайнштейн, [2]), если прообраз $f^{-1}(y)$ каждой точки $y \in f(X)$ бикомпактен.

Бикомпактное отображение $\tilde{f}\colon \tilde{X} \to Y$ называется *бикомпактификацией* непрерывного отображения $f\colon X \to Y$, если X всюду плотно вложено в $\tilde{X}$. На множестве всех бикомпактификаций заданного отображения f можно ввести частичный порядок: для бикомпактификаций $b_1f\colon X_1 \to Y$ и $b_2f\colon X_2 \to Y$ отображения f будем считать $b_1f \leq b_2f$, если существует естественное (т.е. тождественное на множестве X) отображение пространства X_2 на X_1.

Б. А. Пасынков показал, что для каждого тихоновского отображения $f\colon X \to Y$ существует его максимальная бикомпактификация $g\colon Z \to Y$, которую он обозначил символом βf, а пространство Z, где определена эта максимальная бикомпактификация, — символом $\beta_f X$.

Определение 1 [2]**.** Тихоновское отображение $f\colon X \to Y$ называется *суперпаракомпактным*, если для любого замкнутого в $\beta_f X$ множества F, лежащего в $\beta_f X \setminus X$, найдётся конечнокомпонентное покрытие λ пространства X, выкалывающее множество F в $\beta_f X$ (т.е. $F \cap (\cup[\lambda]_{\beta_f X})$).

Для топологического T_1-пространства X через λX обозначим *суперрасширение* пространства X, т.е. множество всех максимальных сцепленных систем замкнутых в X множеств, наделённое Волмэновской топологией.

Пусть $f\colon X \to Y$ — непрерывное отображение T_1-пространств. Тогда для $\xi \in \lambda X$ система $\{[f(F)]_Y : F \in \xi\}$ является максимальной сцепленной системой. Известно, что эта система однозначно достраивается до максимальной сцепленной системы пространства Y, которую обозначим $\lambda(f)(\xi)$. Итак, для отображения $f\colon X \to Y$ определено отображение $\lambda f\colon \lambda X \to \lambda Y$.

Теорема 1. *Тихоновское отображение $\lambda f\colon \lambda X \to \lambda Y$ суперпаракомпактно тогда и только тогда, когда отображение $f\colon X \to Y$ суперпаракомпактно.*

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О подмножествах пространства идемпотентных вероятностных мер и абсолютные ретракты

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Рассмотрим множество $\mathbb{R}_+ = \mathbb{R} \cup \{-\infty\}$ с двумя алгебраическими операциями: сложением $\oplus$ и умножением $\odot$, определёнными следующим образом: $u \oplus v = \max\{u, v\}$ и $u \odot v = u + v$, $u, v \in \mathbb{R}_+$, где $\mathbb{R}$ — множество действительных чисел.

Пусть X — компактное хаусдорфово пространство, $C(X)$ — алгебра непрерывных функций на X с обычными алгебраическими операциями. На $C(X)$ операции $\oplus$ и $\odot$ определим по правилам $\varphi \oplus \psi = \max\{\varphi, \psi\}$ и $\varphi \odot \psi = \varphi + \psi$, где φ, $\psi \in C(X)$.

Напомним, что функционал $\mu\colon C(X) \to \mathbb{R}$ называется [1] *идемпотентной вероятностной мерой* на X, если он обладает следующими свойствами:

(1) $\mu(\lambda_X) = \lambda$ для всех $\lambda \in \mathbb{R}$, где λ_X — постоянная функция;

(2) $\mu(\lambda \odot \varphi) = \lambda \odot \mu(\varphi)$ для всех $\lambda \in \mathbb{R}$ и $\varphi \in C(X)$;

(3) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$ для всех $\varphi, \psi \in C(X)$;

Для компактного хаусдорфово пространства X обозначим через $I(X)$ множество всех идемпотентных вероятностных мер на X. Рассмотрим $I(X)$ как подпространство пространства $\mathbb{R}^{C(X)}$.

Для заданных компактных хаусдорфовых пространств X, Y и непрерывного отображения $f\colon X \to Y$ можно проверить, что естественное отображение $I(f)\colon I(X) \to I(Y)$, определённое по формуле $I(f)(\mu)(\psi) = \mu(\psi \circ f)$, непрерывно. Более того, конструкция I является нормальным функтором. Поэтому для произвольной идемпотентной вероятностной меры $\mu \in I(X)$ можно определить понятие носителя:

$$\operatorname{supp} \mu = \bigcap \left\{A \subset X : \overline{A} = A,\ \ \mu \in I(A)\right\}.$$

Для положительного целого числа n определим следующее множество:

$$I_n(X) = \{\mu \in I(X) :\ \ |\operatorname{supp} \mu| \le n\}.$$

Положим
$$I_\omega(X) = \bigcup_{n=1}^{\infty} I_n(X).$$
Множество $I_\omega(X)$ всюду плотно [1] в $I(X)$. Идемпотентную вероятностную меру $\mu \in I_\omega(X)$ называют *идемпотентной вероятностной мерой с конечным носителем.*

Пусть X и Y — два компакта, лежащие в пространствах M и N соответственно, где $M, N \in \mathcal{AR}$. Последовательность отображений $f_k\colon M \to N$, $k = 1, 2, \ldots$, назовём фундаментальной последовательностью из X в Y, если для каждой окрестности V компакта Y (в N) существует такая окрестность U компакта X (в M), что $f_k|_U = f_{k+1}|_U$ в V почти для всех k. Это значит, что существует такая гомотопия $f_k\colon U \times [0,1] \to V$, что $f_k(x,0) = f_k(x)$ и $f_{k+1}(x,1) = f_{k+1}(x)$ для всех $x \in U$. Эту фундаментальную последовательность будем обозначать через $\{f_k, X, Y\}$ или коротко через f, и будем писать $f\colon X \to Y$. Скажем, что фундаментальная последовательность $f = \{f_k, X, Y\}$ *порождена отображением* $f\colon X \to Y$, если $f_k(x) = f(x)$ для всех $x \in X$ и для всех $k = 1, 2, \ldots$ Скажем, что пространства X и Y *фундаментально эквивалентны*, если существуют такие две фундаментальные последовательности $f\colon X \to Y$ и $g\colon Y \to X$, что $gf = \mathrm{id}_X$ и $fg = \mathrm{id}_Y$.

Если $r\colon X \to F$ — ретракция и существует такая гомотопия $h\colon X \times [0,1] \to F$, что $h(x,0) = x$, $h(x,1) = r(x)$, для всех $x \in X$, то r называют *деформационной ретракцией*, а F — *деформационным ретрактом* пространства X. Деформационная ретракция $r\colon X \to F$ называется *сильной деформационной ретракцией*, если для гомотопии $h\colon X \times [0,1] \to F$ имеем $h(x,t) = x$ для всех $x \in F$ и всех $t \in [0,1]$ [5].

Теорема 1. *Для произвольного бикомпакта X существует непрерывная, открытая, клеточноподобная (т.е. все её слои стягиваемые) ретракция $r_f^x\colon I_\omega(X) \to \delta(X)$.*

Предложение 1. *Для произвольного компакта X подпространство $\delta(X)$ является сильным деформационным ретрактом компакта $I_\omega(X)$.*

Предложение 2. *Пусть X — конечномерный локально связный компакт. Тогда $I_\omega(X) \setminus \delta(X) \in \mathcal{ANR}$.*

Теорема 2. *Пусть $X \in \mathcal{A}(\mathcal{N})\mathcal{R}$ — компакт и $\dim X < \infty$. Тогда $I_\omega(X) \in \mathcal{A}(\mathcal{N})\mathcal{R}$.*

Предложение 3. *Функтор I_ω сохраняет свойство компакта быть $\mathbb{Q}$-многообразием или гильбертовым кирпичом.*

Предложение 4. Функтор I_ω сохраняет свойство слоёв отображения быть $\mathcal{A}(\mathcal{N})\mathcal{R}$-компактом, компактным $\mathbb{Q}$-многообразием и гильбертовым кирпичом (конечной суммой гильбертовых кирпичей).

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Обобщение теорем Вариньона и Виттенбауэра

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Рассмотрим две теоремы классической планиметрии. Пусть $ABCD$ — произвольный четырёхугольник, диагонали которого не являются параллельными и пересекаются в точке O.

Теорема (Вариньон)**.** *Прямые, соединяющие середины сторон $ABCD$, образуют параллелограмм. Если $ABCD$ не является самопересекающимся, то площадь параллелограмма составляет $\frac{1}{2}$ от площади $ABCD$.*

Теорема (Виттенбауэр)**.** *Пусть стороны четырёхугольника $ABCD$ разделены три равные части. Прямые, соединяющие точки разбиения сторон возле вершин, образуют параллелограмм. Если $ABCD$ не является самопересекающимся, тогда площадь параллелограмма составляет $\frac{8}{9}$ от площади $ABCD$.*

Под A_B^λ будем понимать гомотетию точки B с центром A и коэффициентом λ.

Теорема 1. *Прямые* $A_D^\lambda A_B^\lambda$, $B_A^\lambda B_C^\lambda$, $C_B^\lambda C_D^\lambda$, $D_C^\lambda D_A^\lambda$ *образуют параллелограмм. Если* $ABCD$ *не является самопересекающимся, то площадь параллелограма составляет* $2(\lambda - 1)^2$ *от площади* $ABCD$. *Будем его называть гомотетическим параллелограммом* $K^\lambda L^\lambda M^\lambda N^\lambda$.

Теорема 2. *Гомотетические параллелограммы являются перспективными с центром перспективы в* O. *Кроме того, для любых двух параллелограммов справедливо* $\frac{|OK^{\lambda_1}|}{|OK^{\lambda_2}|} = \frac{|\lambda_1 - 1|}{|\lambda_2 - 1|}$.

Свойства типа нормальности вне диагонали

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Пусть $\mathcal{F}$ — нормальный функтор степени ≥ 3, действующий в категории компактов и их непрерывных отображений. Хорошо известна теорема Федорчука о том, что если компакт $\mathcal{F}(X)$ наследственно нормален, то компакт X метризуем. Справедливо следующее обобщение теоремы Федорчука.

Теорема 1. *Если пространство* $\mathcal{F}(X) \setminus X$ *наследственно паранормально, то компакт* X *метризуем.*

Здесь пространство называется *паранормальным* , если для любой счётной дискретной системы замкнутых множеств $\{D_n : n < \omega\}$ найдётся локально конечная система открытых множеств $\{U_n : n < \omega\}$ такая, что для всех $n < \omega$ выполняется $D_n \subset U_n$ и $D_m \cap U_n \neq \emptyset$ в том и только в том случае, когда $D_n = D_m$ [P. Nyikos, *Topol. Proc.*, **9**(2) (1984), p. 367].

В предположении принципа Йенсена А. В. Иванов построил пример компакта X несчётного характера, для которого пространство $\exp_n(X) \setminus X$ нормально для любого $n > 1$. Заметим, что естественное вложение $X \subset \mathcal{F}(X)$ в случае функтора X^n является отождествлением X и диагонали $\Delta \subset X^n$. Говорят, что пространство X удовлетворяет свойству $\mathcal{P}$ *вне диагонали*, если $X^2 \setminus \Delta \in \mathcal{P}$. Следующая теорема доказана в 1990 году А. В. Архангельским и А. П. Комбаровым.

Теорема 2. *Характер нормального вне диагонали компакта счётен.*

Затем Грюнхаге построил пример нормального вне диагонали компакта, не являющегося совершенно нормальным. Отвечая на вопрос Комбарова, Грюнхаге также построил в предположении CH пример неметризуемого совершенно нормального вне диагонали компакта. Заметим, что в известной модели Ларсена и Тодорчевича любой совершенно нормальный вне диагонали компакт метризуем.

Хорошо известно, что нарост $\beta\omega\setminus\omega$ не содержит точек счётного характера.

Теорема 3. *Нарост $\beta\omega\setminus\omega$ является паранормальным вне диагонали компактом.*

Пространство называется *F_σ-паранормальным*, если любое F_σ-подмножество этого пространства паранормально.

Теорема 4. *F_σ-паранормальный вне диагонали компакт содержит всюду плотное множество точек счётного характера.*

Вопрос. Счётен ли характер F_σ-паранормального вне диагонали компакта?

О непрерывной зависимости решений от параметра правой части в условиях типа Каратеодори–Плиша–Дэви

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Мы доказываем непрерывную зависимость решений от параметра α дифференциального включения

$$\dot{y} \in F(t, y, \alpha), \tag{$*$}$$

правая часть которого удовлетворяет условиям типа Каратеодори–Плиша–Дэви. При этом используются методы аксиоматической теории обыкновенных дифференциальных уравнений (см. [В. В. Филиппов, «Пространства решений обыкновенных дифференциальных уравнений», *Изд-во МГУ*, Москва, 1993]).

Теорема. *Пусть для многозначного отображения $F\colon U \times M \to \mathbb{R}^n$:*

1) *существует интегрируемая по Лебегу функция* $\varphi(t)$ *такая, что* $F(t,y,\alpha) \subseteq [O_{\varphi(t)}(\vec{0})]$ *при всех* $(t,y) \in U$ *и* $\alpha \in M$*;*
2) *для любого* $\alpha \in M$ *и любого решения* $z(t)$ *неравенства* $\|\dot{z}(t)\| \leq \varphi(t)$ *почти при всех* t *множество* $F(t,z(t),\alpha)$ *замкнуто и выпукло и отображение* F_t *полунепрерывно сверху в точке* $(z(t),\alpha)$*.*

Тогда решения дифференциального включения $(*)$ *непрерывно зависят от параметра* $\alpha \in M$*.*

В качестве следствия приведём утверждение, обобщающее общеизвестную теорему классической теории обыкновенных дифференциальных уравнений (см. [Ф. Хартман, «Обыкновенные дифференциальные уравнения», *Мир*, Москва, 1970]).

Следствие. *Пусть однозначная функция* $f\colon U \times M \to \mathbb{R}^n$

1) *ограничена по модулю константой* $C \geq 0$*;*
2) *для любого* $\alpha \in M$ *функция* $f(t,y,\alpha)$ *непрерывна по* (t,y) *и для любого* $t \in \mathbb{R}$ *функция* $f(t,y,\alpha)$ *непрерывна по* (y,α)*.*

Тогда решения дифференциального уравнения

$$\dot{y} = f(t,y,\alpha)$$

непрерывно зависят от параметра $\alpha \in M$*.*

Однородные подпространства произведения экстремально несвязных пространств

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В [1] было доказано, что произведение псевдокомпактных групп псевдокомпактно. В [2] исследуются свойства типа компактности произведения однородных пространств. Были построены (Theorem 1.1) однородные псевдокомпактные пространства X, Y, для которых произведение $X \times Y$ не псевдокомпактно. В предположении ZFC+MA построены (Theorem 4.1) однородные счётно компактные пространства X, Y, для которых произведение $X \times Y$ не псевдокомпактно. В [2] был поставлен вопрос (Question 5.1(a)) о таком примере без дополнительных теоритико-множественных предположений.

Теорема 1. *Для любого $p \in \beta\omega \setminus \omega$ существуют однородные пространства X и Y такие, что X p-компактно, Y счётно компактно в счётной степени и $X \times Y$ не псевдокомпактно. Пространство X^τ счётно компактно для любого τ.*

В [2] также поставлен, всё ещё открытый, вопрос (Question 5.3), существует ли однородное псевдокомпактное пространство X, для которого произведение $X \times X$ не псевдокомпактно.

В [8] доказывается, что компактные подмножества экстремально несвязной группы конечны. Z. Frolík [6] доказал, что однородные экстремально несвязные компактные пространства конечны. Оказывается, компактные подмножества однородных экстремально несвязных пространства конечны. Это решает вопросы 4.5.2 и 4.5.3 из [7]. Это утверждение также вытекает из Теоремы 2(c) из [12]. Последнее утверждение можно усилить: компактные подмножества однородных подпространств третьей степени экстремально несвязного пространства конечны. Автору неизвестно, можно ли услить это утверждение для четвёртой степени. В предположении континиум гипотезы CH можно. Более того, в предположении CH компактные подмножества однородных подпространств конечных степеней экстремально несвязного пространства конечны. В предположении CH компактные подмножества однородных подпространств счётной степени экстремально несвязного пространства метризуемы. Неизвестно, можно ли это утверждение доказать наивно.

Получено усиление упомянутой теоремы Фролика: компактные однородные подпространства конечной степени экстремально несвязного пространства конечны.

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О функторах знакопеременных мер

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Исследуются функторы U^ε, где $\varepsilon = \beta, R, \tau$, единичного шара знакопеременных борелевских мер. Показано, что эти функторы удовлетворяют только трём из семи свойств нормальности, присущих функторам вероятностных мер. А именно, эти функторы почти непрерывны, сохраняют отображения с плотными образами и пересечения замкнутых подмножеств нормальных пространств. Кроме того, для бесконечного дискретного пространства X пространства $U^\varepsilon(X)$ не удовлетворяют первой аксиоме счётности и даже не являются пространствами Фреше–Урысона. Отсюда вытекает, что функторы $U^\varepsilon\colon \mathcal{T}ych \to \mathcal{T}ych$ не сохраняют топологические вложения, вес топологических пространств и их метризуемость. Также эти функторы не сохраняют совершенные отображения (даже пространств со счётной базой). Стоит отметить при этом, что в категории $\mathcal{C}omp$ компактных пространств функтор $U = U^\beta = U^R = U^\tau$ обладает всеми свойствами нормального функтора, за исключением свойств сохранения пустого множества, точки и прообразов.

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Понимание топологии

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Основные понятия общей топологии сформировались в рамках развития математического анализа. И сейчас для большинства считающих себя математиками вполне достаточно этого уровня восприятия топологии. Но, сформулированные в общем виде, эти основные понятия общей топологии начали жить и самостоятельной жизнью, находить приложения вне тех рамок, в которых они возникли. Наиболее блестящим примером такого сорта является создание функционального анализа.

Каждый раздел математики определяется набором математических структур, лежащих в его основе. В свою очередь, математические структуры распадаются, в основном, на два типа. Это комбинаторно-алгебраические и тополого-геометрические структуры. Алгебраические структуры начали медленно входить в сознание человека с тех незапамятных времен, когда наш предок понял, что один плюс один это уже два. И если геометрические структуры также имеют глубокую историю, то топологические структуры, как уже было сказано, были введены относительно недавно в связи с развитием математического анализа. Можно уверенно сказать, что есть ещё много мест в математике, где роль топологических связей пока не осознана, не понята.

В докладе будет рассказано, как понимание соответствущей топологической структуры позволило связать в единую цепочку топологические факты теории обыкновенных дифференциальных уравнений и построить на этой основе аксиоматический подход к изложению этой теории, легко покрывающий уравнения с разрывной правой частью и с многозначной правой частью.

Это понимание было основано на опыте работы в топологии в рамках школы А. В. Архангельского. Выработанное тогда геометрическое восприятие топологических структур позволило увидеть, как из них, как из элементов, складывается описание строения пространств решений. Причём сами соответствующие факты теории обыкновеннных дифференциальных уравнений, в основном, уже были замечены теорией. Однако громоздкость и неполнота их описания мешали их правильному пониманию. А правильное понимание дает основу простого, основанного на прозрачной идее, изложения теории.

Уверен, что понимание топологической сути и других разделов математики ещё будет определять их развитие. Но люди, которые будут это делать, должны будут *знать* соответствующие разделы математики и *понимать*, что такое топология.

О представлении, связанном со скрученным

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Пусть G — локально компактная группа с правой мерой Хаара и $\delta(x)$ — её модулярная функция. Для исследования внутреннего представления в $L^2(G)$,

$$\gamma(x)f(y) = \delta(x)^{1/2} f(x^{-1}yx),$$

разумно рассматривать представление

$$\beta(x_1, x_2)f(y) = \delta(x_2)^{1/2} f(x_1^{-1}yx_2)$$

группы $G \times G$ в $L^2(G)$. Поэтому также разумно рассмотреть представление

$$\beta^\phi(x_1, x_2)f(y) = \delta(x_2)^{1/2} f(\phi(x_1^{-1})yx_2)$$

для скрученного представления $\gamma^\phi(x)f(y) = \delta(x)^{1/2} f(\phi(x^{-1})yx)$, где ϕ — некоторый автоморфизм G. Следуя этим путём, сформулируем некоторые результаты для β^ϕ:

Теорема 1. *Если γ не эквивалентно γ^ϕ, то β не эквивалентно β^ϕ.*

Обозначим через $\Gamma(\phi)$ подгруппу в $G \times G$, являющуюся графиком ϕ:

$$\Gamma(\phi) = \{(h, \phi(h)) : h \in G\}.$$

Теорема 2. *Представление* γ *эквивалентно индуцированному представлению* $\mathrm{ind}_{\Gamma(\phi)}^{G\times G} 1_{\Gamma(\phi)}$, *где* $1_{\Gamma(\phi)}$ *— единичное представление группы* $\Gamma(\phi)$.

Пусть ρ_r — правое регулярное представление G и $\int_X F^y\, d\alpha(y)$ — его каноническое разложение по фактор-представлениям.

Теорема 3. *Для каждого* y *существует неприводимое представление* K^y *группы* $G \times G$ *такое, что*

1) *ограничение* K^y *на* $G \times e$ *эквивалентно* $\bar{F}^y \circ \phi$ *и ограничение* K^y *на* $e \times G$ *эквивалентно* F^y, *где* $\bar{F}^y$ *— сопряжённое к* F^y,
2) $\int_X K^y\, d\alpha(y)$ *— каноническое разложение по фактор-представлениям представления* β^ϕ.

Более того, если представление ρ_r *имеет тип* I, *то каждое* F^y *имеет вид* $(\bar{L}^y \circ \phi) \times L^y$ *и* $\int_X (\bar{L}^y \circ \phi) \times L^y\, d\alpha(y)$ *— каноническое разложение по фактор-представлениям представления* β^ϕ.

О линейных гомеоморфизмах пространств непрерывных функций, заданных на разреженных компактах с топологией поточечной сходимости на всюду плотных подмножествах

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Пусть K — регулярное топологическое пространство, $A \subset K$ — всюду плотное подмножество. Через $C_p(A|K)$ обозначим линейное подпространство в $C_p(A)$, состоящее из тех функций $x \in C_p(A)$, для которых $x = y|_A$ для некоторой функции $y \in C_p(K)$. Для данных пространств получена следующая теорема.

Теорема 1. *Пусть* K *— разреженный компакт,* A *и* B *— всюду плотные подмножества в* K *такие, что* $|K \setminus A| = m < n = |K \setminus B|$, $m, n \in \mathbb{N}$. *Тогда пространства* $C_p(A|K)$ *и* $C_p(B|K)$ *не являются линейно гомеоморфными.*

Пусть $s = \prod_{i=1}^{\infty} R_i$, где $R_i = \mathbb{R}$ для всех $i \in \mathbb{N}$ и $c \subset s$ ($c_0 \subset s$) — подпространства всех сходящихся (к нулю) последовательностей.

Следствие 2. *Пространства c и $\underbrace{c \times \ldots \times c}_{n}$, где $n > 1$, негомеоморфны.*

Следствие 3. *Пространства c_0 и c не являются линейно гомеоморфными, но пространства $c_0 \times c$ и c линейно гомеоморфны.*

Следствие 4. *Пространство c дополняемо не вкладывается в пространство c_0, но пространство c_0 дополняемо вкладывается в c.*

Утверждение о том, что пространства c и c_0 не являются линейно гомеоморфными, было доказано в статье [T. Dobrowolski, W. Marciszewski, *Fund. Math.*, **148**(1) (1995)].

Структура некоторых подмножеств пространства идемпотентных вероятностных мер

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Определение пространства идемпотентных вероятностных мер $I(X)$ на компактном хаусдорфовом пространстве X и подпространства $I_\omega(X) \subset I(X)$ идемпотентных вероятностных мер на X с конечным носителем дано в работе А. А. Заитова и А. Я. Ишметова, публикуемой в настоящем сборнике (на с. 74–75).

Заметим, что если μ — идемпотентная вероятностная мера с конечным носителем $\operatorname{supp} \mu = \{x_1, x_2, \ldots, x_k\}$, то μ можно представить в виде $\mu = \lambda_1 \odot \delta_{x_1} \oplus \lambda_2 \odot \delta_{x_2} \oplus \cdots \oplus \lambda_k \odot \delta_{x_k}$ единственным образом, где $-\infty < \lambda_i \leq 0$, $i = 1, 2, \ldots, k$, $\lambda_1 \oplus \lambda_2 \oplus \cdots \oplus \lambda_k = 0$. Здесь, как обычно, для $x \in X$ через δ_x обозначен функционал на $C(X)$, определённый формулой $\delta_x(\varphi) = \varphi(x)$, $\varphi \in C(X)$, и называемый *мерой Дирака, сосредоточенной в точке x*.

Для компакта X и непустого множества $A \subset X$ положим

$$S_I(A) = \{a \in I(X) : \operatorname{supp} a \cap A \neq \emptyset\}.$$

По построению, включение $A \subset B$ влечет $S_I(A) \subset S_I(B)$.

Предложение 1. *Для компакта X, всякого его открытого подмножества U и функтора I имеет место*

$$I(X \setminus U) = I(X) \setminus S_I(U).$$

Следствие 1. *Для всякого открытого подмножества U компакта X множество $S_I(U)$ открыто в $I(X)$.*

Предложение 2. *Для замкнутого подмножества A компакта X множество $S_I(A)$ замкнуто в $I(X)$.*

Теорема 1. *Для произвольного непустого замкнутого подмножества A компакта X, где $A \neq X$, подпространство $I(A)$ есть Z-множество в $I(X)$.*

Теорема 2. *Для произвольного компакта X и каждого натурального $n \in \mathbb{N}$, $n < |X|$, подпространство $I_n(X)$ является Z-множеством в $I(X)$.*

Следовательно, $I_\omega(X)$ является σ-Z-множеством.

Теорема 3. *Для произвольного непустого множества A в X подмножество $S_I(A) = \{\mu \in I(X) : A \cap \operatorname{supp} \mu \neq \emptyset\}$ max-plus выпукло.*

Пример 1. Пусть $n = \{0, 1, 2, \ldots, n-1\}$ — n-точечное дискретное пространство. Для всякого подмножества $A \subset n$ множество $S_I(A)$ открыто в $I\,(n)$. В том числе для каждого $i \in n$ множество $S_I(\{i\})$ открыто. Имеем

$$S_I(\{i\}) = I\,(n) \setminus I\,(\{0, 1, 2, ..., i-1, i+1, ..., n-1\})\,.$$

Пересечение множеств $S_I(\{i\})$ есть внутренность бикомпакта $I\,(n)$, т.е.

$$\bigcap_{i=1}^{n-1} S_I\,(\{i\}) = \operatorname{Int} I\,(n)\,.$$

Отметим, что для подмножеств A и B компакта X имеет место $S_I(A \cap B) \subset S_I(A) \cap S_I(B)$. Но обратное не верно.

Действительно, рассмотрим множества $A = \{0, 1, 2\}$ и $B = \{1, 2, 3\}$. Тогда для $\mu = \lambda_1 \odot \delta_0 \oplus \lambda_2 \odot \delta_3$, где $-\infty < \lambda_1, \lambda_2 \leq 0$ и $\lambda_1 \oplus \lambda_2 = 0$, имеем $\mu \in S_I(A) \cap S_I(B)$, но $\mu \notin S_I(A \cap B)$.

Теорема 4. *Для всякого замкнутого подмножества A компакта X множество $S_I(A)$ есть G_δ-множество в $I(X)$.*

Следствие 2. *Для открытого подмножества U компакта X множество $S_I(U)$ является F_σ-множеством.*

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О группе изометрий слоёных многообразий

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Пусть (M, g) — гладкое риманово многообразие размерности n с римановой метрикой g. Классической задачей римановой геометрии является изучение группы изометрий. Группа изометрий $G(M)$ риманова многообразия на себя образует подгруппу группы $\mathrm{Diff}(M)$ всех диффеоморфизмов M на себя. Как показали S. B. Myers, N. Steenrod [*Ann. Math.*, 1939], группа $G(M)$ с компактно-открытой топологией имеет естественную структуру группы Ли.

Пусть M и N — гладкие многообразия размерности n, на которых заданы гладкие k-мерные слоения F_1 и F_2 соответственно (здесь $0 < k < n$).

Определение 1. Диффеоморфизм $\varphi\colon M \to N$ называется диффеоморфизмом, сохраняющим слоение, если образ $\varphi(L_\alpha)$ любого слоя L_α слоения F_1 является слоем слоения F_2.

В случае, когда $M = N$ и $F_1 = F_2$, говорят о диффеоморфизме слоёного многообразия (M, F).

Определение 2. Диффеоморфизм $\varphi\colon M \to M$ слоёного многообразия называется *изометрией слоёного многообразия* (M, F), если сужение $\varphi\colon L_\alpha \to \varphi(L_\alpha)$ является изометрией.

Обозначим через $G_F^r(M)$ множество всех изометрий класса C^r слоёного многообразия (M, F), где $r \geq 0$. Множество $G_F(M)$ является подмножеством множества всех диффеоморфизмов $\mathrm{Diff}(M)$ многообразия M на себя. Вопросу об изометрических отображениях слоений посвящены работы [А. Я. Нарманов, Д. А. Скоробогатов, *Доклады Акад. Наук Респ. Узбекистан*, 2004], [Д. А. Скоробогатов, *Узбекский матем. журнал*, 2000]. В этих работах изучены вопросы, при каких условиях всякая изометрия слоения является изометрией многообразия.

В настоящей работе исследуется группа $G_F(M)$ с компактно-открытой топологией и топологией, которая будет введена ниже. Эта топология зависит от слоения F и совпадает с компактно-открытой топологией, когда F является n-мерным слоением. Если коразмерность слоения равна n, то сходимость в этой топологии совпадает с поточечной сходимостью.

Пусть $\{K_\lambda\}$ — семейство всех компактных множеств, где каждое K_λ является подмножеством некоторого слоя F и пусть $\{U_\beta\}$ — семейство всех открытых подмножеств M. Рассмотрим для каждой пары $K_\lambda \subset L_\alpha$ и любого U_β совокупность всех отображений $f \in G_F(M)$, для которых $f(K_\lambda) \subset U_\beta$. Эту совокупность отображений будем обозначать через

$$[K_\lambda, U_\beta] = \{f\colon M \to M \mid f(K_\lambda) \subset U_\beta\}.$$

Это семейство образует базу некоторой топологии, которую назовём F-компактно-открытой топологией (слоёной компактно-открытой топологией). Установлено, что имеет место

Теорема 1. *Пусть M — гладкое связное многообразие конечной размерности. Тогда группа его гомеоморфизмов* $\mathrm{Homeo}(M)$ *является топологической группой относительно компактно-открытой топологии. В частности, подгруппы* $\mathrm{Diff}(M), G_F(M)$ *также являются топологическими группами в этой топологии.*

Лемма 2. *Пусть M и B — гладкие римановы многообразие размерностей n и m соответственно, F — слоение, порождённое субмерсией $f\colon M \to B$. Тогда группа $G_F(M)$ содержит отображения, которые не являются элементами группы изометрий $G(M)$ риманова многообразия (M, g).*

Теорема 2. *Группа $G_F(M)$ является топологической группой с F-компактно-открытой топологией.*