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Numerical study for solving time-space fractional Fokker Planck equation with variable coefficients

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Fokker-Planck equation (FPE) is known for modeling various issues in electron relaxation in gases, nucleation, and quantum optics. Scientific appearances such as wave diffusion, constant random motion, DNA, and RNA molecules' biological code, and arrangement materialization are shown by [FPPDEs] with fractional differential functions of time and space as provided in [1]. Such FPPDE applications with time and space-fractional differential equations have involved relevant researchers to address the topic. Brownian motion and the approaches to reaction-kinetics of reactive fluids based on material diffusivity [2] are now being explored in a variety of technologies; physicochemical systems and biological synthesis.

Our reason for researching the problem of variable coefficients is its relevance in applications of light propagation in obstacle-containing tissues. A model for tissues that contain a tumor or another physiological anomaly is this obstacle scattering problem. A crucial step in developing methods for diagnosing tissue health is the recognition of the obstacle scattering problem in tissues. However, due to a lack of computational methods to solve this issue, findings for this issue are minimal. Such findings may be used in other applications as well.

In this work, we consider the following general model of time-space fractional Fokker-Planck equation

$$\frac{\partial^\alpha w(s,t)}{\partial t^\alpha} = -D_s^\beta(E(s,t)w(s,t)) + D_s^{2\beta}(F(s,t)w(s,t)) + g(s,t), 0<\alpha,\beta\le 1 \tag{1}$$

with initial and boundary condition

$$\begin{aligned} &w(s,0) = \varsigma_0(s), && 0\le s\le L \\ &w(0,t)=\sigma_1(t), \quad w(L,t)=\sigma_2(t), && 0\le t\le T \end{aligned} \tag{2}$$

where α and β are parameters describing the order of the fractional time and space derivatives, respectively, $E(s,t), F(s,t)\ge 0$ are diffusion and drift coefficient respectively, $g(s,t)$ is represented sources and sinks, $\dfrac{\partial^\alpha w(s,t)}{\partial t^\alpha}$ is the Caputo fractional derivative of order α. the numerical scheme and alternating direction implicit finite difference scheme is presented for Eq.(1) and Eq.(2) will be in the following form

$$w_i^{k+1}+\sum_{l=0}^{k-1}(\xi_{\alpha,l+1}-\xi_{\alpha,l})w_i^{k-l}+c_1\sum_{l=0}^{i}\eta_{\beta,l}E_{i-l}^{k+1}w_{i-l}^{k+1}-c_2\sum_{l=0}^{i+1}\eta_{2\beta,l}F_{i-l+1}^{k+1}w_{i-l+1}^{k+1}=\xi_{\alpha,k}w_i^0+G_i^{k+1}. \quad (3)$$

Caputo and the Riemann Liouville fractional derivatives are considered in the temporal and spatial directions, respectively. The Riemann Liouville derivative is approximated by the standard Grunwald approximation and shifted Grunwald approximation (the Caputo derivative with order $0 < \alpha < 1$ and the Riemann Liouville derivative for two orders $\beta, 2\beta, 0.5 < \beta < 1$. The submitted numerical scheme (3) for solving (2) and (2) is unconditional stability and convergence with temporal order of convergent $2-\alpha$ for every space positive definite monotone increasing functions $E(s,t)$ and space positive definite monotone decreasing convex functions $F(s,t)$, we will give a numerical example to verify the theoretical analysis as in Fig.1.

References

1) Heinsalu E., Patriarca M., Goychuk I., Schmid G., Hanggi P. Fractional Fokker-Planck dynamics: Numerical algorithm and simulations. Phys. Rev. E. 4-73 2006.
2) Benson D.A., Wheatcraft S.W., Meerschaert M.M. Application of a fractional advection-dispersion equation, Water Resour. Res.36 (6) 1403–1412 2000.
3) Zhuang P., Liu F., Anh V., Turner I. New solution and analytical techniques of the implicit numerical method for the anomalous subdiffusion equation. SIAM J. Numer. Anal.46, 1079–1095 2008.

Illustrations

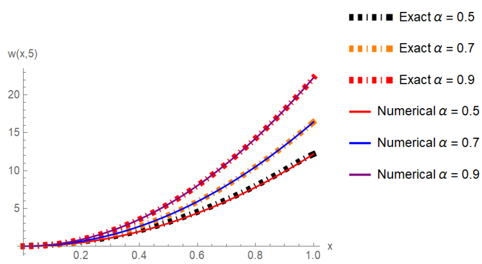

Рис. 1. Graphs of solutions to the corresponding problem for various values of the order of fractional caputo derivative α for example with exact solution $s^2(1+t^{1+\alpha})$