Секция «Математика и механика»

On the lattice of all soluble Fischer classes

Царев Александр Александрович
Кандидат наук
Витебский государственный университет, МФ, Витебск, Беларусь
E-mail: alex_ vitebsk@mail.ru

All groups considered are finite and soluble. All unexplained notations and terminology are standard (see [1]).

The symbol $\mathfrak{N}$ denotes the class of all nilpotent groups. Recall that for any class of groups $\mathfrak{F} \supseteq (1)$ an intersection of all such normal subgroups N of a group G that $G/N \in \mathfrak{F}$ is called an $\mathfrak{F}$-residual of G.

A closure operation on the set of classes of groups is a map c with the properties:

$$\mathrm{c}\mathfrak{X} \subseteq \mathrm{c}\mathfrak{Y} \text{ if } \mathfrak{X} \subseteq \mathfrak{Y} \quad \text{and} \quad \mathfrak{X} \subseteq \mathrm{c}\mathfrak{X} = \mathrm{c}(\mathrm{c}\mathfrak{X}).$$

A class of groups $\mathfrak{X}$ is called c-closed if $\mathfrak{X} = \mathrm{c}\mathfrak{X}$.

Let $\mathfrak{X}$ be a class of groups. Then

$\mathrm{N}_0\mathfrak{X} = (G \mid \exists N_i \lhd\lhd G,\ N_i \in \mathfrak{X}$ where $i = 1, ..., r$ and $G = \langle N_1, ..., N_r \rangle)$;

$\mathrm{S}_\mathrm{F}\mathfrak{X} = (G \mid G \leq H \in \mathfrak{X}$ and $G^{\mathfrak{N}} \lhd\lhd H)$.

The class of groups $\mathfrak{X} \neq \varnothing$ is called a *Fischer class* if $\mathfrak{X} = \mathrm{S}_\mathrm{F}\mathfrak{X}$ and $\mathfrak{X} = \mathrm{N}_0\mathfrak{X}$.

A lattice L is said to be an *algebraic lattice* if it is a complete lattice and every element of L can be written as a join of compact elements. With respect to inclusion $\subseteq$ the set of all soluble Fischer classes is a complete lattice.

Theorem. *The lattice of all soluble Fischer classes is algebraic.*

Литература

1. Doerk K., Hawkes T. Finite soluble groups – Berlin – New York: Walter de Gruyter, 1992. – 891 p.